ST. HUGH'S CATHOLIC PRIMARY VOLUNTARY ACADEMY

MATHEMATICS CALCULATION POLICY

CALCULATION GUIDANCE taken from "Calculation Guidance: NCETM October 2015"

The following key areas are priorities for our school in Mathematics.

We develop children's fluency with basic number facts

Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. A short time everyday should be spent on these basic facts and quickly leads to improved fluency. This can be done using simple whole class chorus chanting. The evidence suggests that this is not meaningless rote learning; rather, this is an important step to developing conceptual understanding through identifying patterns and relationships between the tables (for example, that the products in the 6× table are double the products in the 3× table). This helps children develop a strong sense of number relationships, an important prerequisite for procedural fluency.

A suggested order for learning x-tables is as follows to provide opportunities to make connections:

×10	×5	×2	×4	×8	×3	×6	×9	×7

We develop children's fluency in mental calculation

Efficiency in calculation requires having a variety of mental strategies. It is crucial to emphasise the importance of 10 and partitioning numbers to bridge through 10. For example:

9 + 6 = 9 + 1 + 5 = 10 + 5 = 15.

It is helpful to make a 10 as this makes the calculation easier.

We develop fluency in the use of formal written methods

Teaching column methods for calculation provides the opportunity to develop both procedural and conceptual fluency. Children can understand the structure of the mathematics presented in algorithms, with a particular emphasis on place value. Base ten apparatus should be used and illustrated in books to support the development of fluency and understanding.

Informal methods of recording calculations are an important stage to help children develop fluency with formal methods of recording. They are stepping stones to formal written methods.

We develop children's understanding of the = symbol

The symbol **=** is an assertion of equivalence. If we write:

3 + 4 = 6 + 1

then we are saying that what is on the left of the = symbol is necessarily equivalent to what is on the right of the symbol. But many children interpret = as being simply an instruction to evaluate a calculation, as a result of always seeing it used thus:

3 + 4 =

5 × 7 =

16 – 9 =

If children only think of = as meaning "work out the answer to this calculation" then they are likely to

get confused by empty box questions such as:

$$3 + \square = 8$$

Later they are very likely to struggle with even simple algebraic equations, such as:

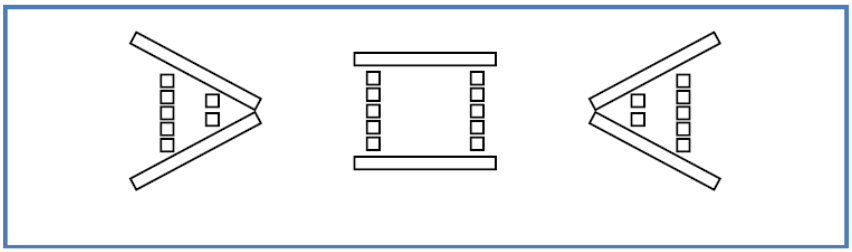
$$3y = 18$$

One way to model equivalence such as 2 + 3 = 5 is to use balance scales.

We should aim to vary the position of the = symbol and include empty box problems from Year 2 onwards to deepen children's understanding of the = symbol.

Teach inequality alongside teaching equality

To help young children develop their understanding of equality, they also need to develop understanding of inequality. One way to introduce the < and > signs is to use rods and cubes to make a concrete and visual representations such as:



to show that 5 > 2), 5 is equal is less than 5 (2 is greater than 2 (5 to 5 (5 = 5), and 2 < 5).

Balance scales can also be used to represent inequality.

Incorporating both equality and inequality into examples and exercises can help children develop their conceptual understanding. For example, in this empty box problem children have to decide whether the missing symbol is <, = or >:

$$5 + 7 \square 5 + 6$$

An activity like this also encourages children to develop their mathematical reasoning: "I know that 7 is greater than 6, so 5 plus 7 must be greater than 5 plus 6".

Asking children to decide if number sentences are true or false also helps develop mathematical reasoning. For example, in discussing this statement:

$$4 + 6 + 8 > 3 + 7 + 9$$

a child might reason that "4 plus 6 and 3 plus 7 are both 10. But 8 is less than 9. Therefore 4 + 6 + 8 must be less than 3 + 7 + 9, not more than 3 + 7 + 9".

In both these examples the numbers have been deliberately chosen to allow the children to establish the answer without actually needing to do the computation. This emphasises further the importance of mathematical reasoning.

We have an emphasis on calculation

Young children benefit from being helped at an early stage to start calculating, rather than relying on 'counting on' as a way of calculating. For example, with a sum such as:

4 + 7 =

Rather than starting at 4 and counting on 7, children could use their knowledge and bridge to 10 to deduce that because 4 + 6 = 10, so 4 + 7 must equal 11.

Look for pattern and make connections

We aim to use a great many visual representations of the mathematics and some concrete resources. Understanding, however, does not happen automatically, children need to reason by and with themselves and make their own connections. Throughout the school, children will be encouraged to look for pattern and connections in the mathematics. The question "What's the same, what's different?" is used frequently to make comparisons. For example *"What's the same, what's different between the three times table and the six times table?"*

We use intelligent practice

In designing exercises for children, teachers are advised to create an appropriate path for practicing the thinking process with increasing creativity (Gu, 1991). This will allow children to develop both procedural and conceptual fluency. Children are required to reason and make connections between calculations. The connections made improve their fluency.

2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

Shanghai Textbook Grade 2 (aged 7/8)

We use empty box problems

Empty box problems are a powerful way to help children develop a strong sense of number through intelligent practice. They provide the opportunity for reasoning and finding easy ways to calculate. They enable children to practise procedures, whilst at the same time thinking about conceptual connections.

A sequence of examples such as

3 + ☐ = 8

3 + ☐ = 9

$$3 + \square = 10$$

$$3 + \square = 11$$

helps children develop their understanding that the = symbol is an assertion of equivalence, and invites children to spot the pattern and use this to work out the answers.

This sequence of examples does the same at a deeper level:

$$3 \times \square + 2 = 20$$

$$3 \times \square + 2 = 23$$

$$3 \times \square + 2 = 26$$

$$3 \times \square + 2 = 29$$

$$3 \times \square + 2 = 35$$

Children should also be given examples where the empty box represents the operation, for example:

$$4 \times 5 = 10 \square 10$$

$$6 \square 5 = 15 + 15$$

$$6 \square 5 = 20 \square 10$$

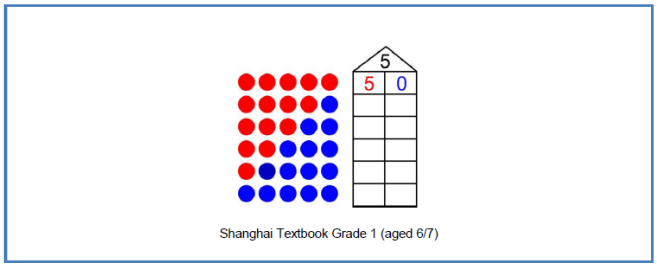
$$8 \square 5 = 20 \square 20$$

$$8 \square 5 = 60 \square 20$$

These examples also illustrate the careful use of variation to help children develop both procedural and conceptual fluency.

We expose mathematical structure and work systematically

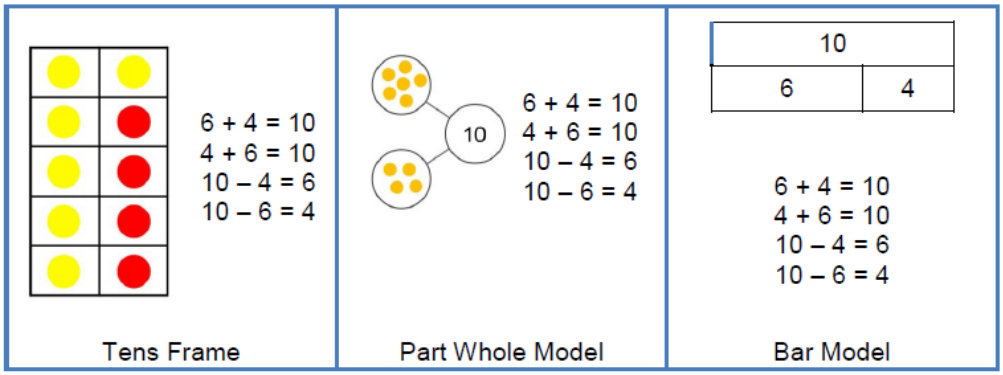
Developing instant recall alongside conceptual understanding of number bonds to 10 is important. This can be supported through the use of images such as the example illustrated below:



Shanghai Textbook Grade 1 (aged 6/7)

The image lends itself to seeing pattern and working systematically and children can connect one number fact to another and be certain when they have found all the bonds to 5.

Using other structured models such as tens frames, part whole models or bar models can help children to reason about mathematical relationships.



Connections between these models should be made, so that children understand the same mathematics is represented in different ways. Asking the question “What’s the same what’s different?” has the potential for children to draw out the connections.

Illustrating that the same structure can be applied to any numbers helps children to generalise mathematical ideas and build from the simple to more complex numbers, recognising that the structure stays the same; it is only the numbers that change. For example:

10	
6	4

6 + 4 = 10
4 + 6 = 10
10 – 6 = 4
10 – 4 = 6

247	
173	74

173 + 174 = 247
74 + 173 = 247
247 – 173 = 74
247 – 74 = 173

6.2	
3.4	2.8

3.4 + 2.8 = 6.2
2.8 + 3.4 = 6.2
6.2 – 3.4 = 2.8
6.2 – 2.8 = 3.4

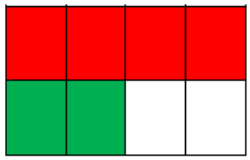
We move between the concrete and the abstract

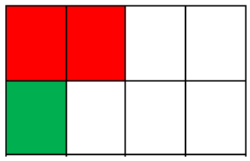
Children’s conceptual understanding and fluency is strengthened if they experience concrete, visual and abstract representations of a concept during a lesson. Moving between the concrete and the abstract helps children to connect abstract symbols with familiar contexts, thus providing the opportunity to make sense of, and develop fluency in the use of, abstract symbols.

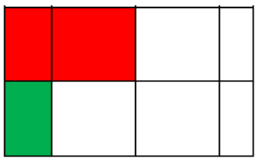
For example, in a lesson about addition of fractions children could be asked to draw a picture to represent the sum

$$\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

Alternatively, or in a subsequent lesson, they could be asked to discuss which of three visual images correctly represents the sum, and to explain their reasoning:







We contextualise the mathematics

A lesson about addition and subtraction could start with this contextual story:

“There are 11 people on a bus. At the next stop 4 people get on. At the next stop 6 people get off. How many are now on the bus?”

This helps children develop their understanding of the concepts of addition and subtraction. But during the lesson the teacher should keep returning to the story. For example, if the children are thinking about this calculation

14 – 8

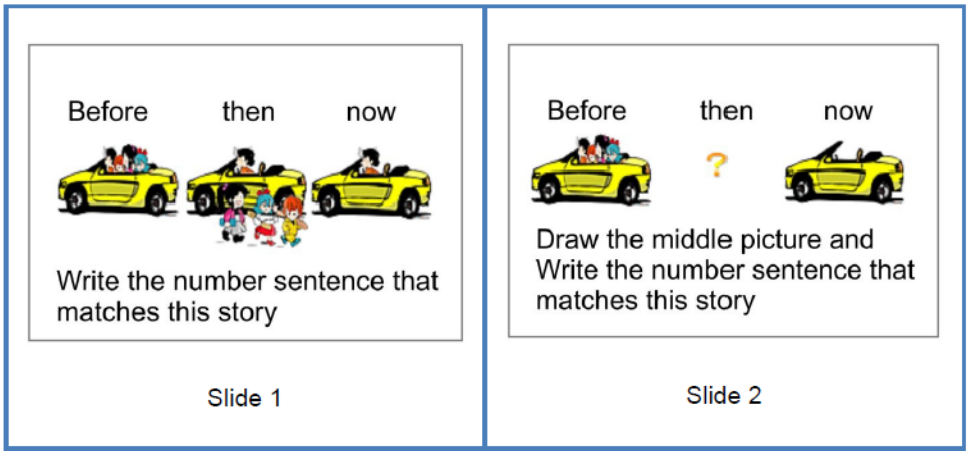
then the teacher should ask the children:

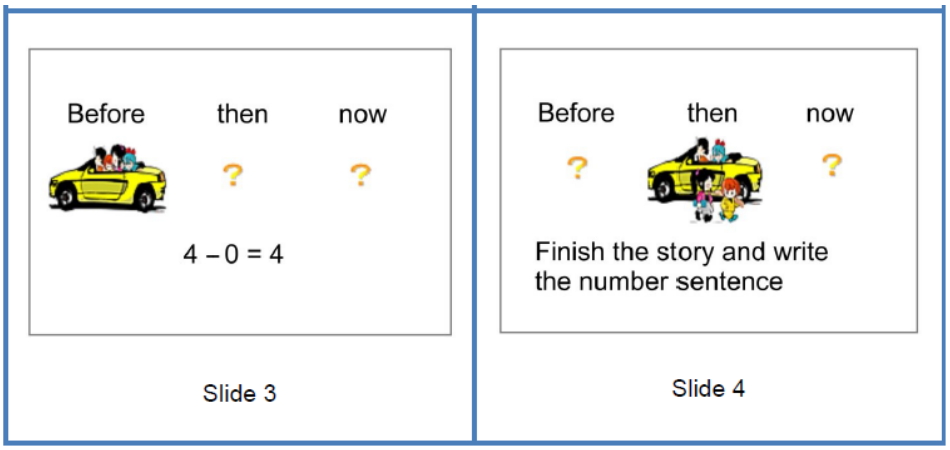
“What does the 14 mean? What does the 8 mean?”, expecting that children will answer:

“There were 14 people on the bus, and 8 is the number who got off.”

Then asking the children to interpret the meaning of the terms in a sum such as 7 + 7 = 14 will give a good assessment of the depth of their conceptual understanding and their ability to link the concrete and abstract representations of mathematics

The four slides below are taken from a lesson delivered by one of the Shanghai teachers (Li Dong)





Notice how each activity varies. The children are asked to:

Slide 1: Start with the story (concrete) and write the number sentence (abstract).

Slide 2: Start with the story (concrete) and complete it. Then write the number sentence (abstract).

Slide 3: Start with the number sentence (abstract) and complete the story (concrete).

Slide 4: Start with part of the story, complete two elements of it (concrete with challenge) and then write the number sentence (abstract).

The children move between the concrete and the abstract and back to the concrete, with an increasing level of difficulty.

We use questioning to develop mathematical reasoning

Teachers' questions in mathematics lessons are often asked in order to find out whether children can give the right answer to a calculation or a problem. But in order to develop children's conceptual understanding and fluency there needs to be a strong and consistent focus on questioning that encourages and develops their mathematical reasoning.

This can be done simply by asking children to explain how they worked out a calculation or solved a problem, and to compare and contrast different methods that are described. Children should quickly come to expect that they need to explain and justify their mathematical reasoning, and they should soon start to do so automatically – and enthusiastically. Some calculation strategies are more efficient; teachers should scaffold children's thinking to guide them to the most efficient methods, whilst at the same time valuing their own ideas.

Rich questioning strategies include:

- *"What's the same, what's different?"*

In this sequence of expressions, what stays the same each time and what's different?

23 + 10	23 + 20	23 + 30	23 + 40

Discussion of the variation in these examples can help children to identify the relationship between the calculations and hence to use the pattern to calculate the answers.

- *"Odd one out"*

Which is the odd one out in this list of numbers: 24, 15, 16 and 22?

This encourages children to apply their existing conceptual understanding.

Possible answers could be:

“15 is the odd one out because it’s the only odd number in the list.”

“16 is the odd one out because it’s the only square number in the list.”

“22 is the odd one out because it’s the only number in the list with exactly four factors.”

If children are asked to identify an ‘odd one out’ in this list of products:

24 × 3 36 × 4 13 × 5 32 × 2

they might suggest:

“36 × 4 is the only product whose answer is greater than 100.”

“13 × 5 is the only product whose answer is an odd number.”

- *“Here’s the answer. What could the question have been?”*

Children are asked to suggest possible questions that have a given answer. For example, in a lesson about addition of fractions, children could be asked to suggest possible ways to complete this sum:

$$\square + \square = \frac{3}{4}$$

- *Identify the correct question*

Here children are required to select the correct question:

A 3.5m plank of wood weighs 4.2 kg

The calculation was:

3.5 ÷ 4.2

Was the question:

a. How heavy is 1m of wood?

b. How long is 1kg of wood?

- *True or False*

Children are given a series of equations are asked whether they are true or false:

4 × 6 = 23 4 × 6 = 6 × 4 12 ÷ 2 = 24 ÷ 4 12 × 2 = 24 × 4

Children are expected to reason about the relationships within the calculations rather than calculate.

- *Greater than, less than or equal to >, <, or =*

These **3.4 × 1.2 ○ 3.4 5.76 ○ 5.76 ÷ 0.4 4.69 × 0.1 ○ 4.69 ÷ 10** types of questions are further examples of intelligent practice where conceptual understanding is developed alongside the development of procedural fluency. They also give pupils who are, to use Ofsted’s phrase, *rapid graspers* the opportunity to apply their understanding in more complex ways.

We expect children to use correct mathematical terminology and to express their reasoning in complete sentences

The quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to use correct mathematical terminology (e.g. saying 'digit' rather than 'number') and to explain their mathematical thinking in complete sentences.

I say, you say, you say, you say, we all say

This technique enables the teacher to provide a sentence stem for children to communicate their ideas with mathematical precision and clarity. These sentence structures often express key conceptual ideas or generalities and provide a framework to embed conceptual knowledge and build understanding. For example:

If the rectangle is the whole, the shaded part is one third of the whole.

Having modelled the sentence, the teacher then asks individual children to repeat this, before asking the whole class to chorus chant the sentence. This provides children with a valuable sentence for talking about fractions. Repeated use helps to embed key conceptual knowledge.

Another example is where children fill in the missing parts of a sentence; varying the parts but keeping the sentence stem the same. For example:

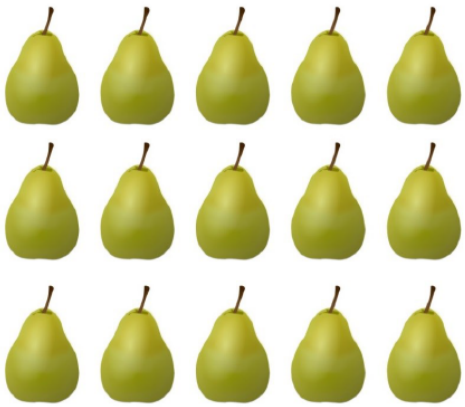
There are 12 **stars**. $\frac{1}{3}$ of the **stars** is equal to **4 stars**

Children use the same sentence stem to express other relationships. For example:

There are 12 **stars**. $\frac{1}{4}$ of the **stars** is equal to **3 stars**

There are 12 **stars**. $\frac{1}{2}$ of the **stars** is equal to **6 stars**

Similarly:



There are **15 pears**. $\frac{1}{3}$ of the **pears** is equal to **5 pears**

There are **15 pears**. $\frac{1}{5}$ of the **pears** is equal to **3 pears**

When talking about fractions it is important to make reference to the whole and the part of the whole in the same sentence. The above examples help children to get into the habit of doing so.

Another example is where a mathematical generalisation or "*rule"* emerges within a lesson. For example:

When adding 10 to a number, the ones digit stays the same

This is repeated in chorus using the same sentence, which helps to embed the concept.

We identify difficult points

Difficult points need to be identified and anticipated when lessons are being designed and these need to be an explicit part of the teaching, rather than the teacher just responding to children's difficulties if they happen to arise in the lesson. The teacher should be actively seeking to uncover possible difficulties because if one child has a difficulty it is likely that others will have a similar difficulty. Difficult points also give an opportunity to reinforce that we learn most by working on and through ideas with which we are not fully secure or confident. Discussion about difficult points can be stimulated by asking children to share thoughts about their own examples when these show errors arising from insufficient understanding. For example:

$$\frac{2}{14} - \frac{1}{7} = \frac{1}{7}$$

FORMAL CALCULATION PROGRESSION

YEAR 2

- Column and addition & subtraction introduced in basic form.
- Addition may introduce carrying to next column as exchange (showing written and working alongside stage 4 and stage 5 addition)
- Subtraction working into stage 4 and stage 5 .

YEAR 3

- Column and addition & subtraction formalised including borrowing and carrying.
- Children should be moving into stage 5 and stage 6 addition and subtraction. (Some children may still require the less abstract stage 4 if understanding not shown)
- Multiplication & Division into stage 4 and stage 5.
- Multiplication methods for partitioning and grid method are introduced alongside practical arrays.

YEAR 4

- Column and addition & subtraction including larger numbers/decimals; children should be confident with stage 6 addition and subtraction.
- Multiplication into stage 5/6/7, continuing with grid method and introducing column multiplication.
- Division into stage 5/6. Short division method is introduced.
- Remainders may be introduced dependant on understanding.

YEAR 5

- Column and addition & subtraction secure, linking to range of mathematical concepts.
- Multiplication stage 7/8; column method for multiplication used (up to 4 digits multiplied by 1 or 2 digits)
- Division stage 7/8. Short division used confidently. Long division (chunking) introduced.
- Contextual understanding of remainders is required.
- Multiply fractions by whole numbers.

YEAR 6

- Column and addition & subtraction secure, linking to range of mathematical concepts.
- Multiplication stage 8; column method for multiplication used confidently.
- Division stage 8. Both short division and long division (chunking) used with confidence.
- Contextual understanding of remainders is secure.
- Multiplication/division of fractions.
- Multiplication of decimals.

Teachers should ensure that children who do not understand new calculation stages go back to secure and understand previous stages of development before moving on.

MATHEMATICAL VOCABULARY AND SPELLING

KS1

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at Key Stage 1.

YEAR 3/4

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

YEAR 5/6

Pupils should read, spell and pronounce mathematical vocabulary correctly.

Mathematical vocabulary should be included in spelling practice and may be tested.

Glossary of terms

Cardinal number	The number of items in a set, the quantity but not the order of things. For example, 'There are five pencils in a pot.'
Conservation of number	If a group of objects is rearranged, the total number of objects stays the same.
Consecutive	Following in order. Consecutive numbers are adjacent in a count. For example, 5, 6, 7 are consecutive numbers. 25, 30, 35 are consecutive multiples of 5.
Commutativity	For addition and multiplication, the numbers in a calculation can be done in any order and will result in the same answer. E.g. 3 x 4 = 12 and 4 x 3 = 12 or 3 + 4 = 7 and 4 + 3 = 7. Addition and multiplication and communicative. Subtraction and Division are not communicative. However, children must understand that the numbers in a calculation can also be in any order but will result in a different answer. E.g. 7 – 5 = 2 and 5 – 7 =-2.
Digit	One of the symbols of a number system, most commonly the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. For example, the number 29 is a two-digit number;5 is a one-digit number. The position or place of a digit in a number conveys its value.
Dividend	The quantity which is to be divided. E.g. in the calculation, 12 ÷ 3, the dividend is 12.
Divisor	The quantity by which another quantity is to be divided. E.g. for the calculation, 12 ÷ 3, the divisor is 3.
Estimate	**Verb:** To arrive at a rough or approximate answer **Noun:** A rough or approximate answer
Fewer	Used to compare two or more sets of countable (discrete) objects. For example, 'There are fewer apples in this bag.'
Less	Used to compare 'uncountable' (continuous) quantities including measures. For example, 'This bottle has less water in it than that one'.
Long Multiplication	A formal calculation strategy that builds on understanding of the grid method into a compact column method. The multiplier is larger than 12 and therefore is partitioned during the process to aid calculation. Long multiplication is a multi-stage calculation which requires a final addition calculation in order to reach the final outcome.
Inverse of multiplication(as a method of division)	Counting up from 0 in multiples to reach a number in order to solve a division calculation. Inverse of multiplication is used to see how many amounts make a given number. E.g. starting at 0 and counting up in steps of 3 until 12 is reached. Some children find counting on in the multiples from 0 easier that repeated subtraction and this is fine so long as they understand they are using the inverse of multiplication rather than repeated subtraction
Number line	A line on which numbers are represented by points. Division marks are numbers, rather than spaces. They begin at any number and extend into negative numbers. They can show any number sequence. 012345678910
Number track	A numbered track along which counters may be moved. The number in a region represents the number of single moves from the start. Each number occupies a cell and is used to number the cell Numbers may have a matching illustration Supports learning to read numbers in numerals Supports locating ordered numbers They should start at 1 and not 0.
Numeral	A symbol used to denote a number. For example, 5, 23 and the Roman V are all numbers written in numerals.
Ordinal numbers	A term that describes a position within an ordered set. For example, first, second, third, fourth…twentieth.
Partition	To separate a set into subsets To split a number into component parts. For example, the two-digit number 38 can be partitioned into 30 + 8 or 19 + 19.

Pattern	A systematic arrangement of numbers, shapes or other elements according to a rule.
Principle of Exchange	The naming system when counting collections, that as soon as we have a group of ten we call them something else. The number we call ten (10 in numerals) is the most important in our naming system. E.g. ten ones are called one ten, ten tens are called one hundred; ten hundreds are called one thousand.
Proportionally	The relationship of one thing to another in terms of quantity, size, or number/out of the whole/2 out of 5. Proportionally puts the emphasis on the relationship rather than the quantity
Quotient	The result of a division calculation. E.g. In the calculation of 12 ÷ 3, the quotient is 4.
Ratio	The comparison of two properties /2:3. All ratio relationships are proportional
Repeated subtraction	Repeatedly subtracting the same amount each time in order to solve a division calculation. The idea of repeated subtraction should be 'how many times can I take ____ away from ____? E.g. 12 ÷ 3 using repeated subtraction we should start as 12 and repeatedly count down in steps of 3 until 0 is reached.
Representation	The wide variety of ways to capture an abstract mathematical concept or relationship. This may be visible, such as a number sentence, a display of manipulative materials, or a graph, but it may also be an internal way of seeing and thinking about mathematical idea. Regardless of their form, representations can enhance students' communication, reasoning, and problem-solving abilities; help them make connections among ideas; and aid them in learning new concepts and procedures.
Sequence	An ordered set of numbers or shapes arranged according to a rule
Short Multiplication	A formal calculation strategy that builds on understanding of the grid method into a compact column method. The multiplier is 12 or less and therefore is not partitioned during the process as the calculations should rely on knowledge of key multiplication facts up to 12 x 12. An expanded short multiplication method details each stage in brackets and shows clear connections to the grid method. This will be used as a vital stage in bridging the understanding from the grid method to short multiplication.
Subitising	This is the process whereby we recognise the size of a set, its cardinality, from the pattern or structure without having to count the number of objects. For example, recognising there are five dots in this pattern.
Zero	Nought or nothing In a place-value system, a place-holder. For example, 105. The cardinal number of an empty set.

Stage 1

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

+ + + + + + + + Addition + + + + + + + +

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will use practical equipment to combine groups of objects to find a total. Practical resources will support children's development of mental pictures and images.

Children will begin to understand commutativity and the principle of exchange. They will be confident in using the terms 'worth' and 'value' when talking about single-digit numbers.

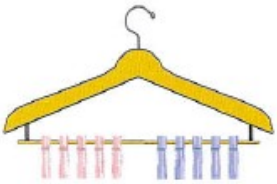
Children can represent calculations using objects and talk about their representations.

Fingers

6 = 1 + 5 1 + 5 = 6

6 – 1 = 5 6 – 5 = 1

Beads or any object



Pegs

Counters

Cubes

Cuisenaire Rods

Numicon

Straws

Stage 2

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

+ + + + + + + + + Addition + + + + + + + + +

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

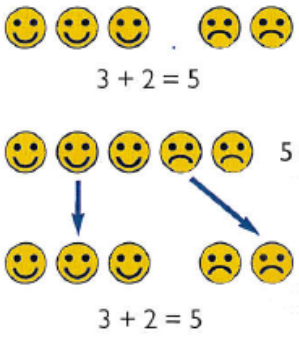
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

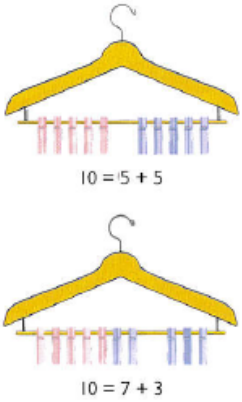
Practical resources will continue to support children's development of mental pictures and images. As these become firm, children will begin to develop ways to represent their mental images and their practical resources using pictures.

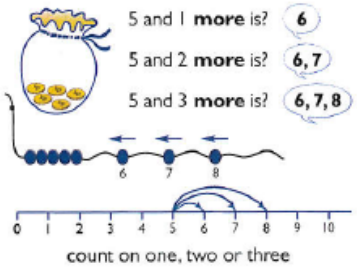
The children will begin to use number sentences alongside their pictures and practical resources.

They will also begin to think and talk flexibly about addition.

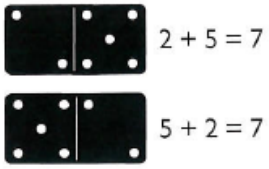
The direct link between addition and subtraction should be made explicit when using models and representations.

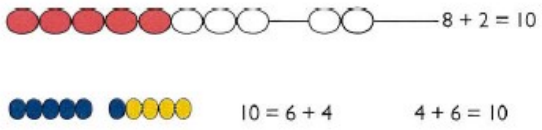


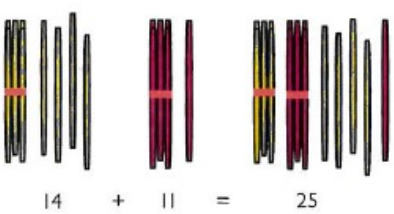


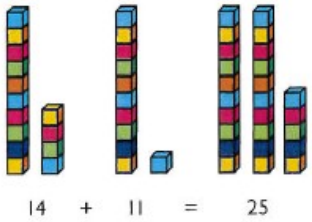


9 and 1 more is 10
9 add 1 equals 10
9 + 1 = 10









Stage 3

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

+ + + + + + + + + Addition + + + + + + + + +

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

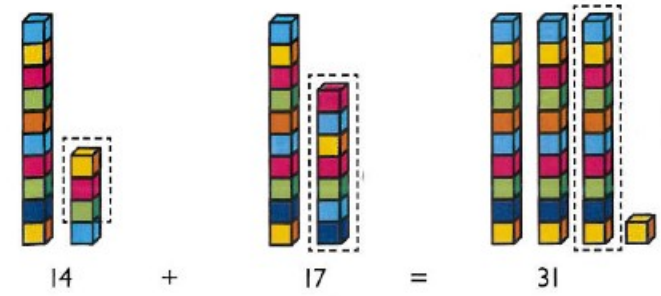
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

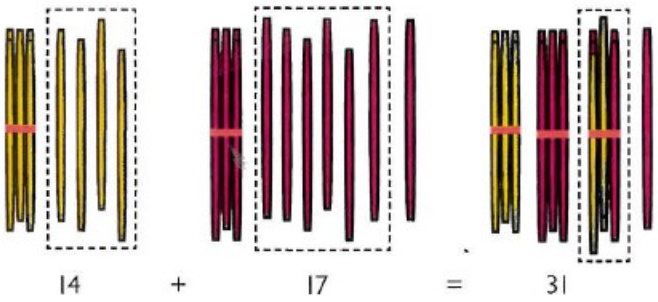
Children will now be confident in using concrete equipment to help them combine groups of objects with numbers up to 20.

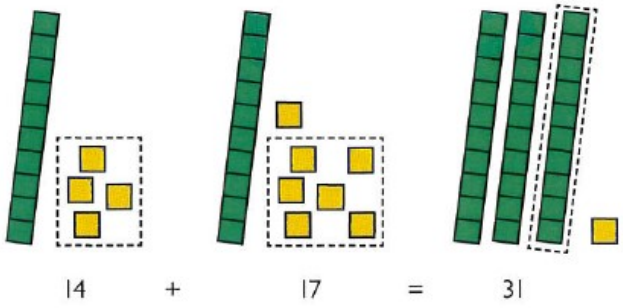
They will continue using practical equipment as they begin to also use **number tracks**, **number lines** and hundred squares to support their mental methods.

Children will start to work with totals greater than 20 which require them to apply their knowledge of the **principle of exchange**. They will talk confidently about this.

14 + 17







Stage 4

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

+ + + + + + + + + Addition + + + + + + + + +

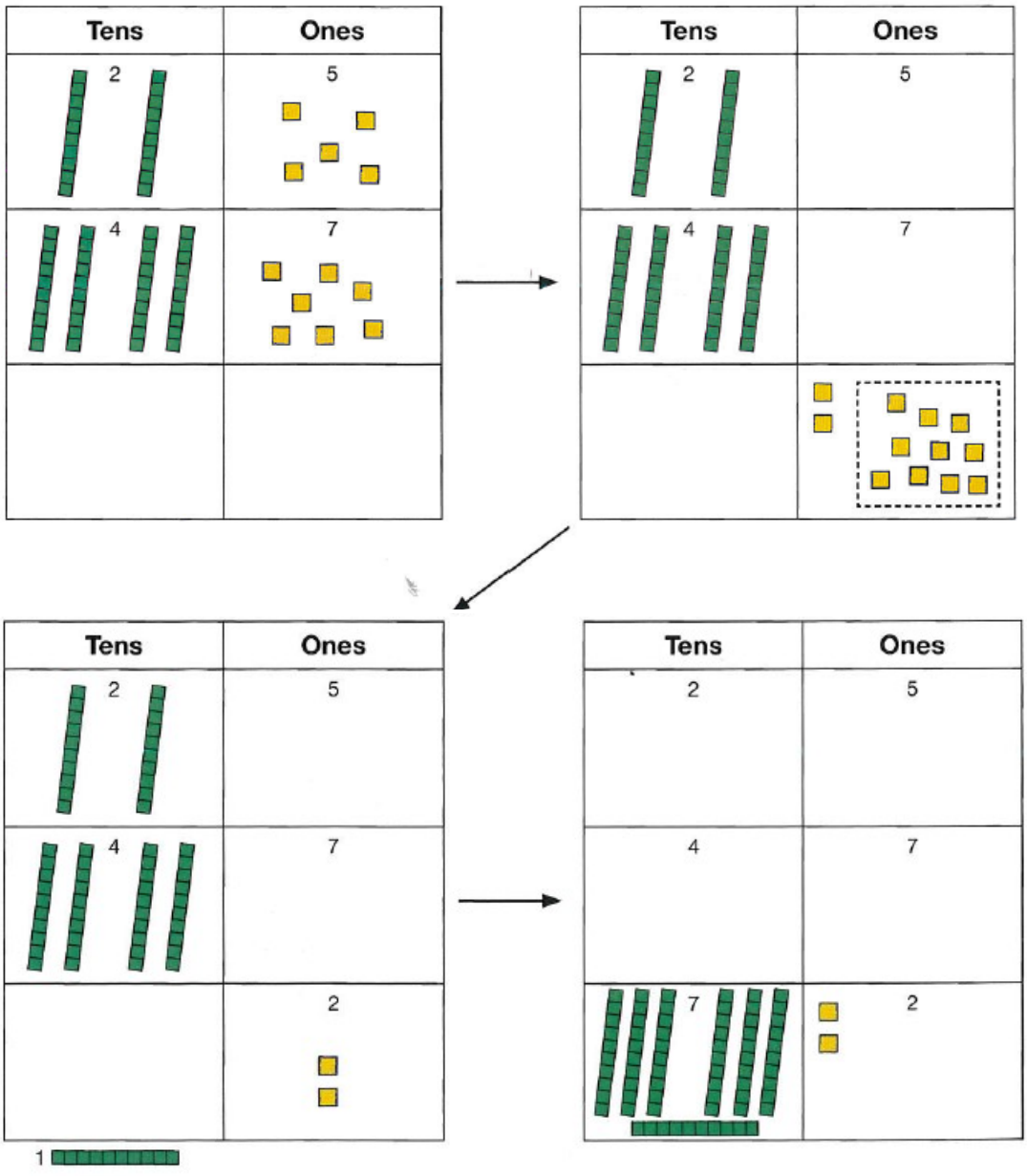
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children are now confident in using concrete equipment to combine objects using the principle of exchange appropriately.

They will now begin to organise their concrete equipment (e.g. Straws, Dienes, Place Value Counters) in a vertical manner where their combined totals are situated at the bottom.

25 + 47



12 ones exchanged to 1 ten and 2 ones

Stage 5

POINTS TO REMEMBER

- Use the language **'calculation'** not 'sum' ('sum' means 'plus or 'total'.)
- Use the language **'digit'** not number (number is the amount or quantity)

+ + + + + + + + + Addition + + + + + + + + +

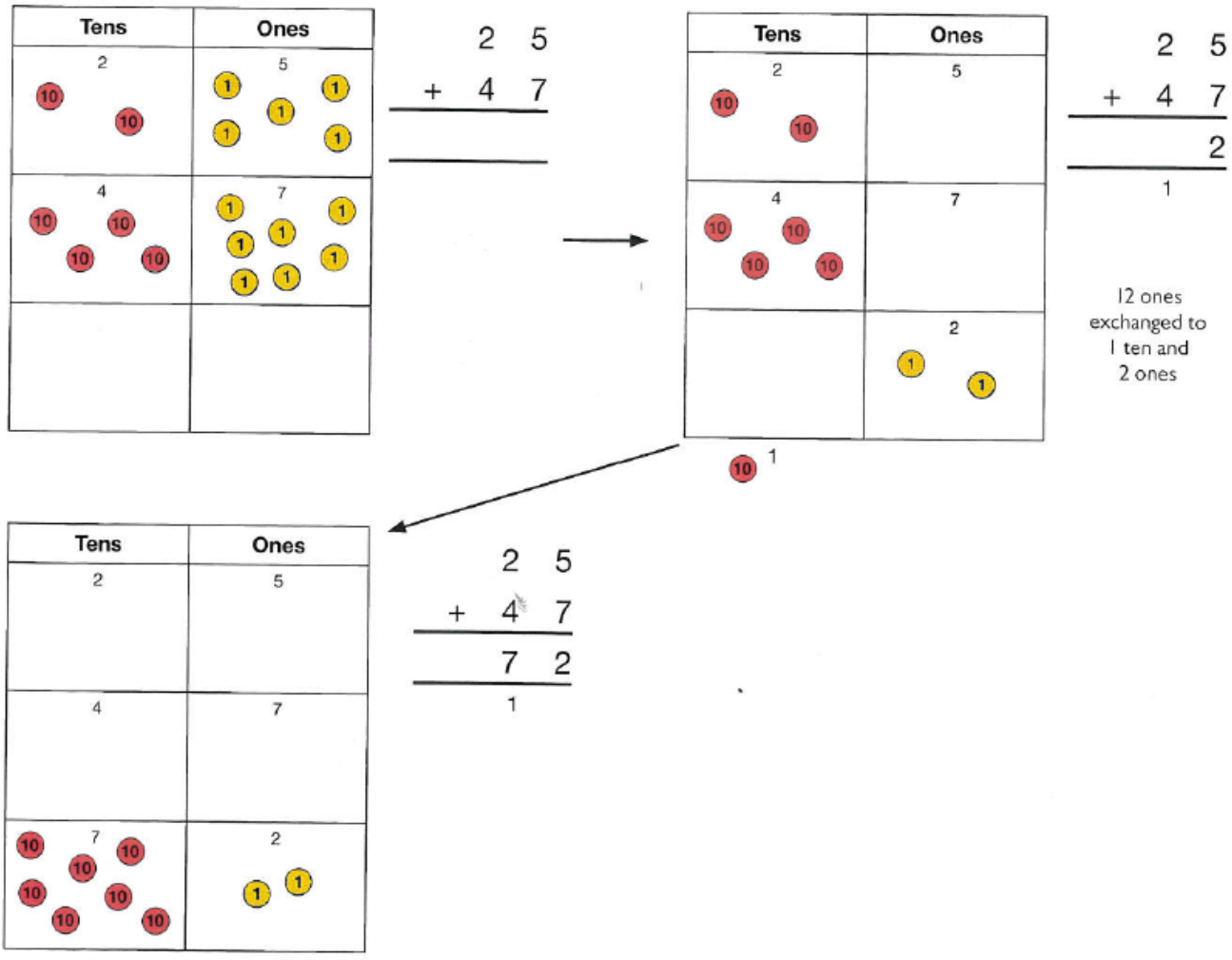
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will now be secure in organising their concrete equipment in a vertical manner where their combined totals are situated at the bottom.

They will be now able to make the links between this representation and the formal column addition when seen alongside each other.

25 + 47



Stage 6

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

+ + + + + + + + Addition + + + + + + + +

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, units/ones boundary, tenths boundary, inverse, how many more to make...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will have a full understanding of the links between the concrete representation for column addition and the formal written method.

They will now be able to explore calculating with larger numbers using their understanding of the formal written method.

$$\begin{array}{rrrr} & 3 & 2 & 7 \\ + & 4 & 9 & 6 \\ \hline & 8 & 2 & 3 \\ \hline & 1 & 1 & \end{array}$$

Calculating with decimals

When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

Stage 1

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

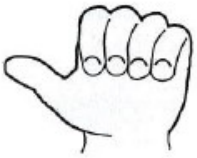
– – – – – – – – Subtraction – – – – – – – –

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

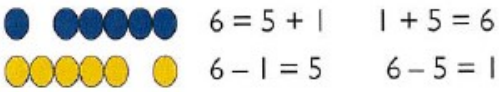
subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than...?, how much more/less is...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will use practical equipment to physically remove an amount from the group to find the total remaining. Practical resources will support children's development of mental pictures and images.

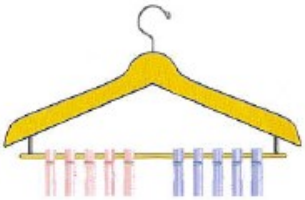
Children can represent calculations using objects and talk about their representations.



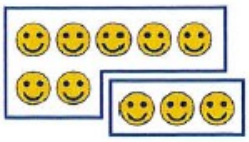
Fingers



Beads or any object



Pegs



Counters

Cubes

Cuisenaire Rods

Numicon

Straws

Children will also be introduced to the language of comparison including equal use of the vocabulary '**less**' and 'more'.

There are more blue than red.
There are less red than blue.

Cubes

Stage 2

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

Subtraction

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is.../than...?, how much more/less is...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Practical resources will continue to support children's development of mental pictures and images. As these become firm, children will begin to develop ways to represent their mental images and their practical resources using pictures.

The children will begin to use number sentences alongside their pictures and practical resources.

They will also begin to think and talk flexibly about subtraction and make links to the inverse of addition.

Children will understand that subtraction is not commutative and so the numbers in a calculation can be in any order but will result in a different answer.

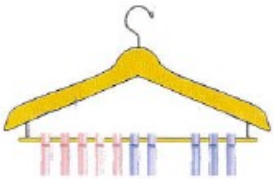
The direct link between addition and subtraction should be made explicit when using models and representations.

$6 + ? = 10$

$10 - 6 = ?$

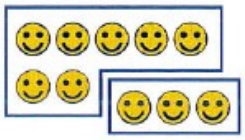
$? + 6 = 10$

$10 - 4 = 6$



$10 - 7 = 3$

$10 - 3 = 7$



$10 - 7 = 3$

$10 - 3 = 7$

$6 - 2 = 4$

Children will continue to be introduced to the language of comparison and its link to finding the difference structure of subtraction.

There are more blue than red.
There are less red than blue.
There are 9 more blue than red.
There are 9 less red than blue.

Stage 3

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

Subtraction

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

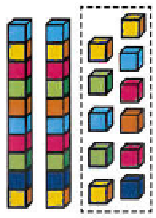
subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than...?, how much more/less is...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will now be confident in using concrete equipment to help them 'take away' and 'find the difference'.

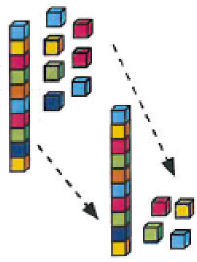
They will continue using practical equipment as they begin to also use number tracks, number lines and hundred squares to support their mental methods.

Children will start to work with numbers greater than 20 which require them to apply their knowledge of the principle of exchange. They will talk confidently about this.

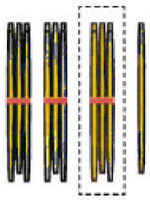
31 – 14

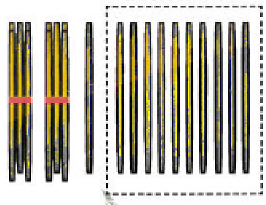


31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with 14

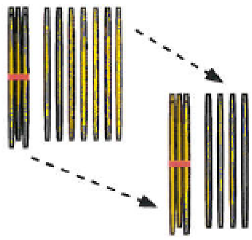


14 can now be removed from the 31 leaving 17

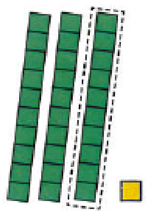


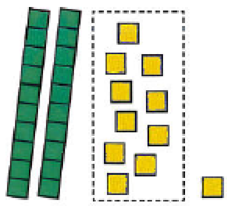


31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with 14

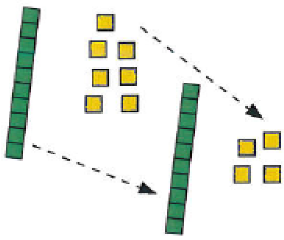


14 can now be removed from the 31 leaving 17





31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with 14



14 can now be removed from the 31 leaving 17

As children become accustomed to repartitioning numbers, they can be introduced to formal notation of the repartitioning.

$^{2}\not{3}\ ^{1}1$

"This is now 20 and 11."

Stage 4

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

Subtraction

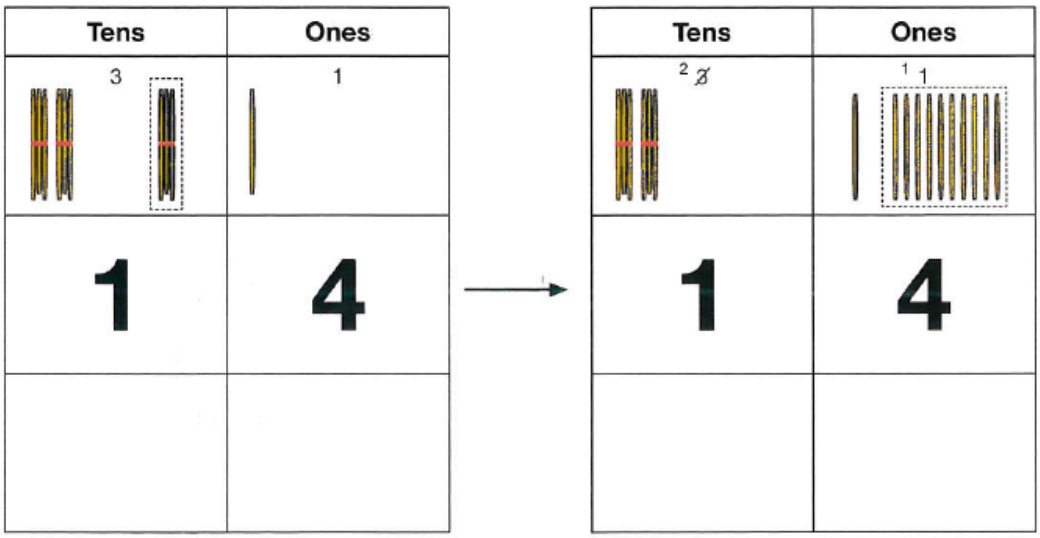
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than...?, how much more/less is...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

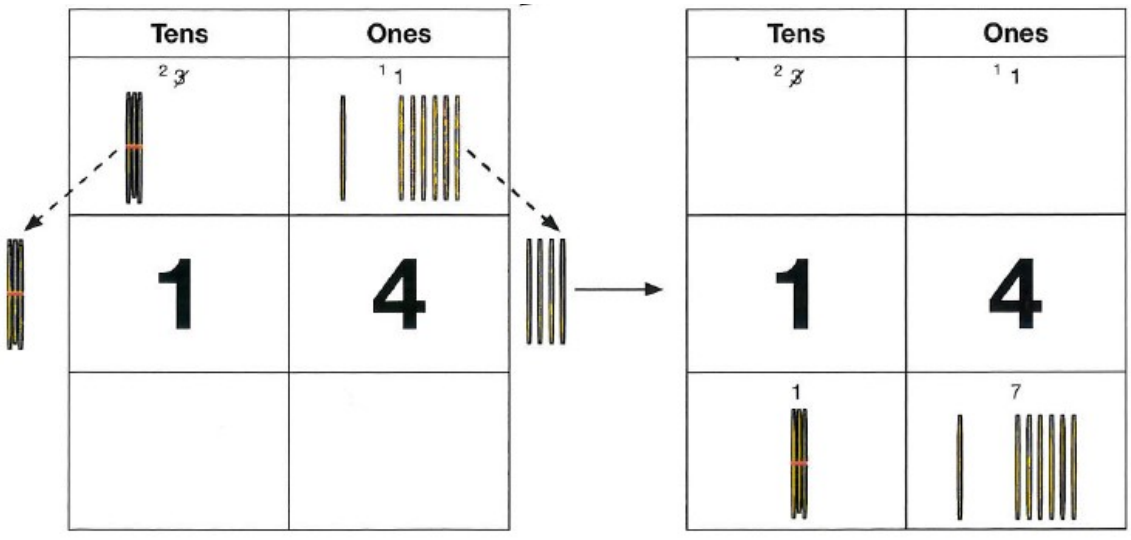
Children are now confident in using concrete equipment to 'take away' and 'find the difference' using the principle of exchange appropriately.

They will now begin to organise their concrete equipment (e.g. Straws, Dienes, Place Value Counters) in a vertical manner where the amount that remains at the end of the calculation is situated at the bottom.

31 – 14



31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with 14



14 can now be removed from the 31

The remaining equipment can then be slid down to the answer box showing what is left

Stage 5

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

Subtraction

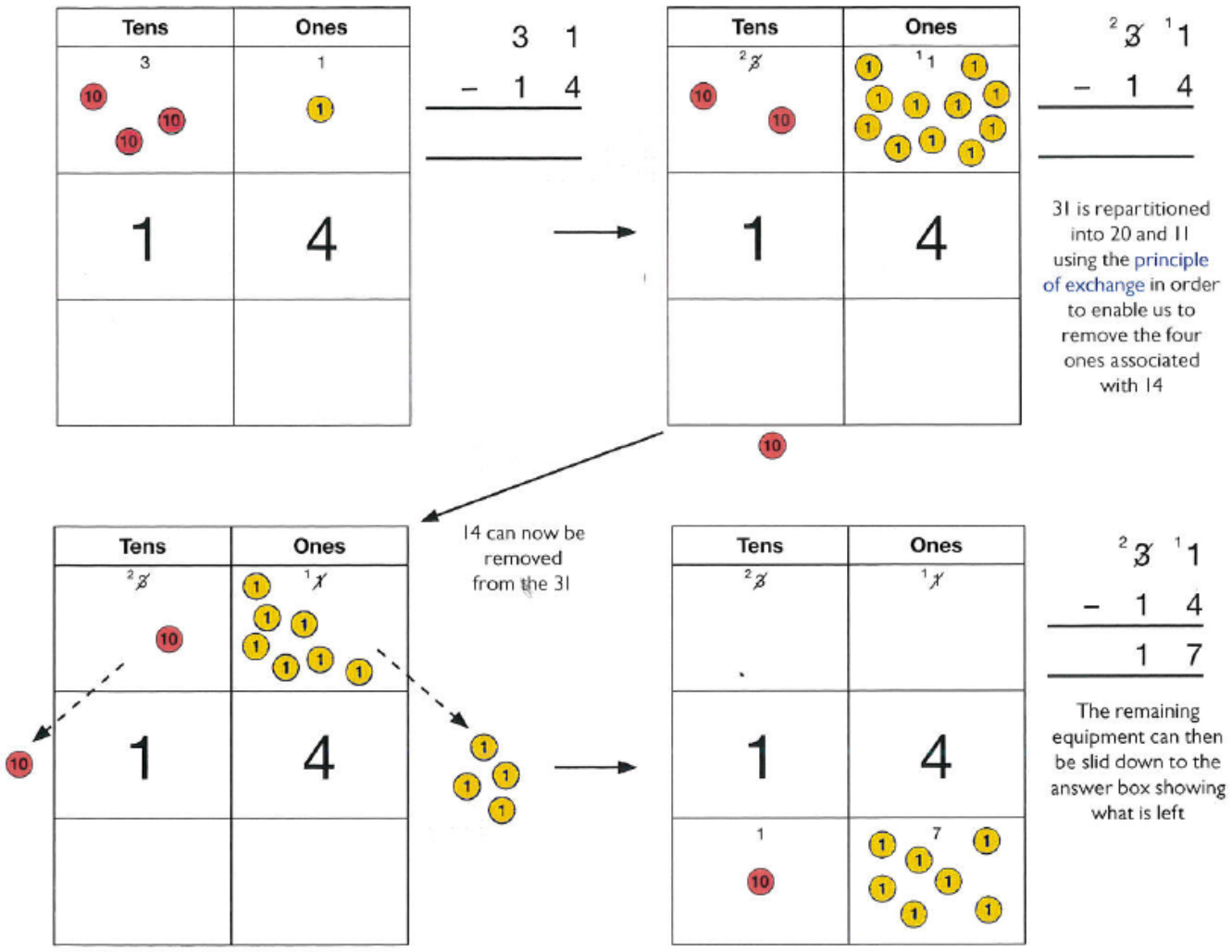
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than...?, how much more/less is...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will now be secure in organising their concrete equipment in a vertical manner for subtraction using the principle of exchange appropriately.

They will be now able to make the links between this representation and the formal column subtraction when seen alongside each other.

31 – 14



Stage 6

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

Subtraction

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is../than...?, how much more/less is...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will have a full understanding of the links between the concrete representation for column subtraction and the formal written method.

They will now be able to explore calculating with larger numbers using their understanding of the formal written method.

$$\begin{array}{rrrr} & 7 & {}^{7}\not{8} & {}^{1}4 \\ - & 2 & 5 & 9 \\ \hline & 5 & 2 & 5 \\ \hline \end{array}$$

Calculating with decimals

When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

Stage 1

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

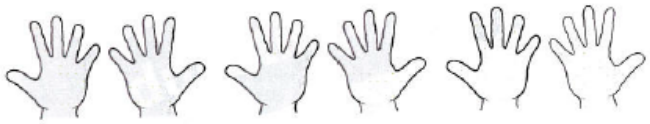
counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____ lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

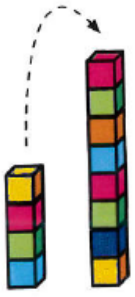
Children will experience practical opportunities involving equal sets or groups using a wide variety of equipment. Practical resources will support children's development of mental pictures and images.

Children will begin to orally count in different multiples including twos, fives and tens making links to natural groupings (e.g. pairs of socks, legs on animals) and the practical resources used.

Children can begin to recognise and continue patterns of multiples using a range of practical resources, e.g. threading beads with two of each colour.

They will begin to use the language and associated representations of doubling.





Double 4 is 8

Stage 2

POINTS TO REMEMBER

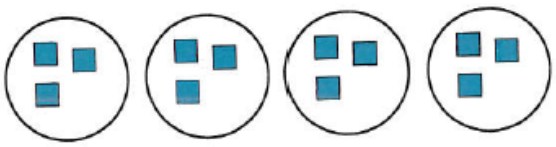
- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

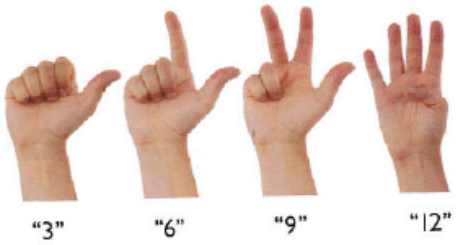
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

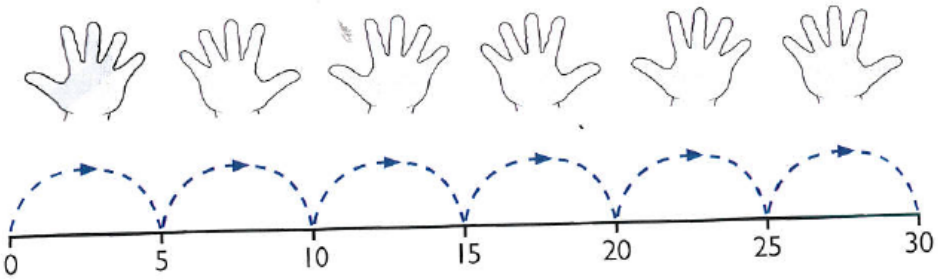
counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

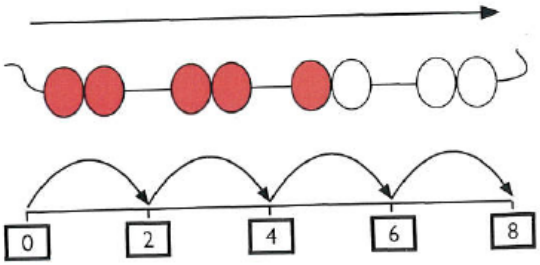
Children will begin to arrange objects into equal groups to aid counting.



They will continue to count in multiples and begin to relate this to multiplication through finger counting.

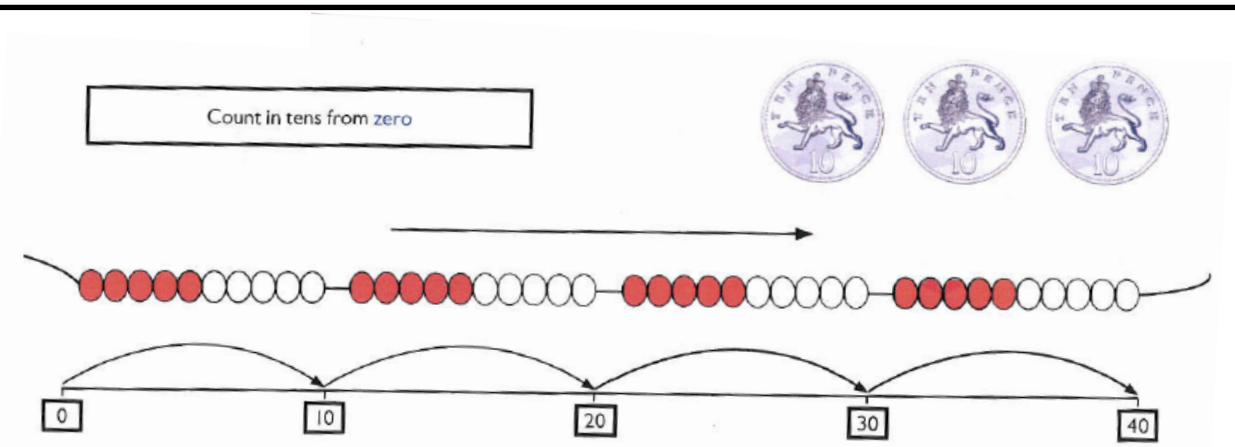




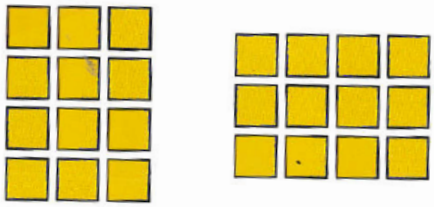


Count in twos from zero

Stage 2 (cont.)



Children will be introduced to the array, using concrete equipment, for small numbers as a way of organising groups to show repeated addition and **commutativity**. They should explore arrays in the world around us, e.g. egg boxes, baking trays, wrapping papers; and use them to answer questions such as 'How many eggs would we need to fill the egg box?' 'How do you know?'



Stage 3

POINTS TO REMEMBER

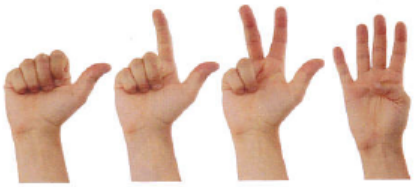
- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

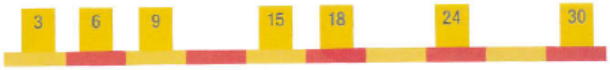
VOCABULARY **Ensure the correct vocabulary is used at all stages of learning**

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

Children will continue to count in multiples and relate this to multiplication through finger counting.

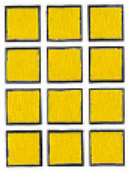


"7" "14" "21" "28"



They will be able to model a calculation using a practical array which demonstrates an effective method of counting and the link to repeated addition. Children need to explore related multiplication facts of a given number by making a variety of arrays and explaining what they show.

Representing 12



3 x 4 = 12
4 x 3 = 12

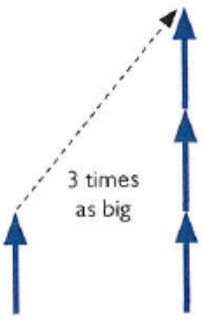
2 x 6 = 12
6 x 2 = 12

1 x 12 = 12
12 x 1 = 12

3 + 3 + 3 + 3 + 3 = 15

5 + 5 + 5 = 15

The children should be confident with their use of the language of scaling when talking about multiplication.



Stage 4

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

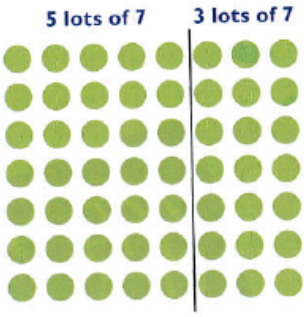
x x x x x x x x Multiplication x x x x x x x x

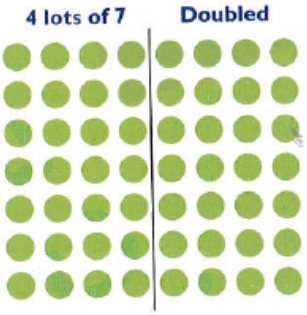
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

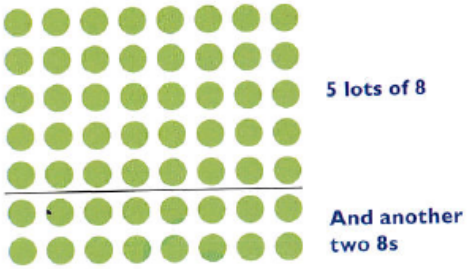
counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

Children will explore practical arrays for larger numbers. They will think flexibly when working with arrays and will be encouraged to look at arrays beyond repeated addition. They will look for 'friendly' numbers to help them efficiently calculate totals within arrays. E.g. for 7x8... Children may find counting in 7s or 8s tricky but they can look for 'friendly' numbers which are easier to calculate e.g. 4x5, 4x2, 4x5, 4x2.

Thinking flexibly about 7 x 8







Children should continue to experience the language of scaling (e.g. scaling up pictures by multiplying by powers of 10, multiplying by powers of 1000 in converting between units of measure)

Stage 5

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

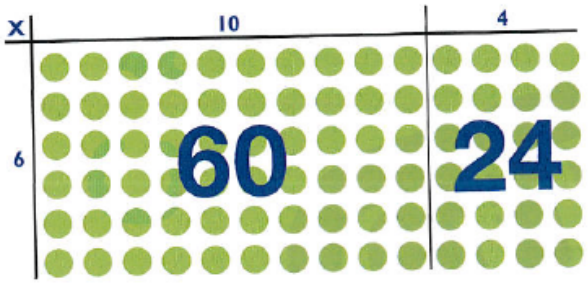
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, ___ times as big, count in ones, count in ___, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

Children will continue to work with arrays, exploring larger numbers, leading into the grid method of multiplication. Practical experiences may still be required for some children as they enter this stage. To begin with, children should see the array with the grid lines. When appropriate, children should move to using the grid displaying the numbers only.

Children should begin using grid method for 2- and 3- digit by 1 digit numbers and should be given the chance to relate this to facts they know about arrays where needed.

Throughout this stage, children should be encouraged to estimate an approximate answer in order to check for reasonableness and this should become standard practice.



$(6 \times 10) + (6 \times 4)$

$60 + 24$

84

x	10	4
6	60	24

Stage 6

POINTS TO REMEMBER

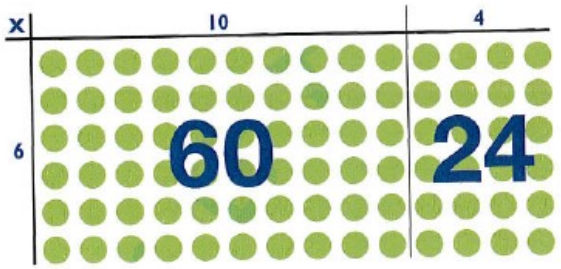
- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

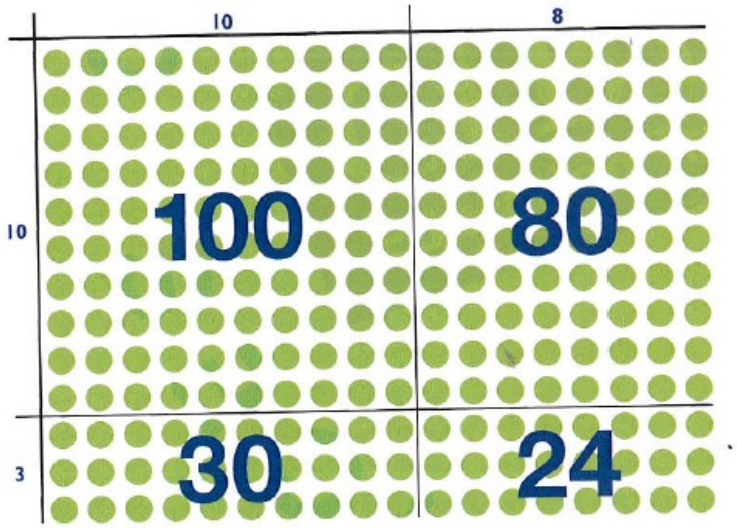
counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

Children will now be secure in using the grid method for multiplying by one-**digit** numbers and will begin to explore the links between the grid method and the expanded method of **short multiplication**.



$$\begin{array}{rr} & 14 \\ \times & 6 \\ \hline & 24 \quad (6 \times 4) \\ + & 60 \quad (6 \times 10) \\ \hline & 84 \end{array}$$

Children will also begin to explore the use of arrays and the grid method for multiplying by two-**digit** numbers.



x	10	8
10	100	80
3	30	24

$$\begin{array}{rrrr} & 1 & 8 & 0 \\ + & & 5 & 4 \\ \hline & 2 & 3 & 4 \\ \hline & 1 & & \end{array}$$

Stage 7

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

Children will now have a good understanding of the expanded **short multiplication** method and will begin to represent this as compact **short multiplication** for TU x U.

```
  1 4
x   6
-----
  2 4   (6 x 4)
+ 6 0   (6 x 10)
-----
  8 4
```

→

```
  1 4
x   6
-----
  8 4
  2
```

Children will be secure in using the grid method for multiplying by two-**digit** numbers and will begin to explore the links between the grid method and the expanded method of **long multiplication.**

x	10	8
10	100	80
3	30	24

→

```
    1 8
  x 1 3
-------
    2 4   (3 x 8)
    3 0   (3 x 10)
    8 0   (10 x 8)
+ 1 0 0   (10 x 10)
-------
  2 3 4
-------
  1
```

Stage 8

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total')
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

VOCABULARY **Ensure the correct vocabulary is used at all stages of learning**

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

Children will now have a good understanding of the **short multiplication** method.

Children will now have a good understanding of the expanded **long multiplication** method and will begin to represent this as compact **long multiplication**.

```
    1 8
  x 1 3
  -----
    2 4   (3 x 8)
    3 0   (3 x 10)
    8 0   (10 x 8)
+ 1 0 0   (10 x 10)
  -----
  2 3 4
  -----
    1
```

→

```
      2
    1 8
  x 1 3
  -----
    5 4
    3 0
+ 1 8 0
  -----
  2 3 4
  -----
    1
```

Calculating with decimals

When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

Stage 1

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

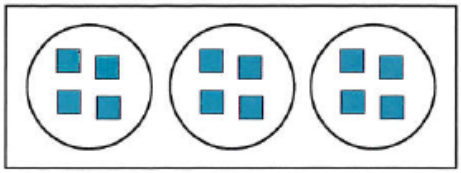
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____ , ____ equal groups

Children will explore the language of sharing. Children will experience practical activities in 'sharing' objects between a small number of groups/people with the emphasis on sharing equally.

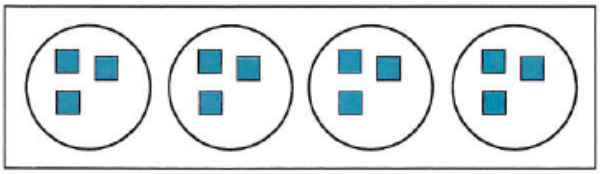
Alongside this, with equal weighting, children should be introduced to 'grouping' objects as a **representation** of division (e.g. 'each person gets 2') with the emphasis on equal groups.

They will begin to use the language and associated **representations** of halving.

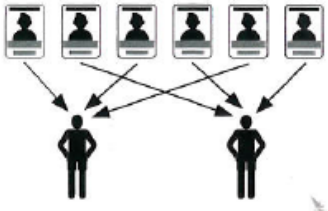
Children can be encouraged to develop ways of recording their findings using pictures.



12 shared into 3 equal groups.
12 shared equally into groups of 4.

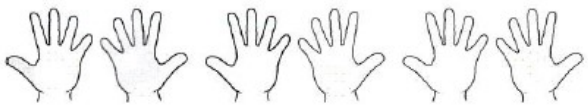


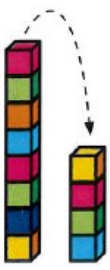
12 shared into 4 equal groups.
12 shared equally into groups of 3.



6 football stickers shared between 2 people

6 football stickers, how many people can have 2 each?





Half 8 is 4

Stage 2

POINTS TO REMEMBER

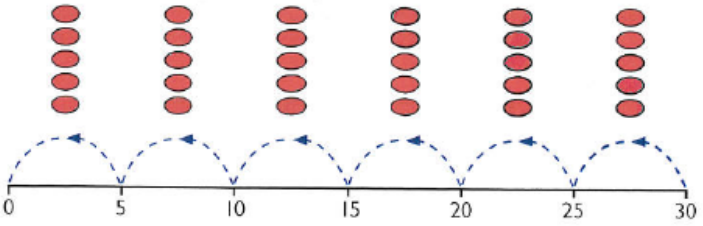
- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

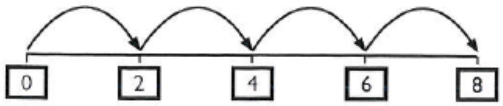
÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

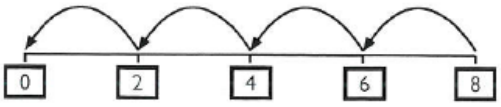
halve, share, share equally, one each, two each, three each..., divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___ , ___ equal groups

Children will relate the grouping of objects to **repeated subtraction** and begin to represent this using a **number line** whilst continuing to use concrete equipment.





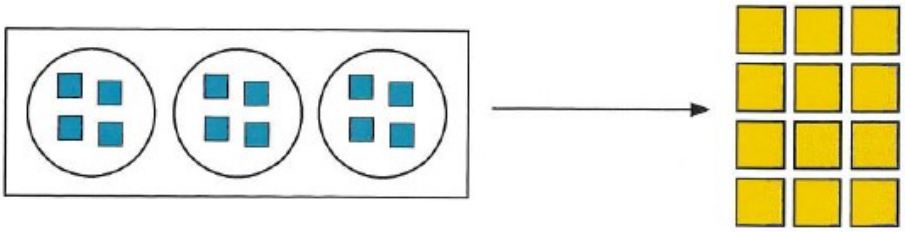
How many 2s are in 8?



How many 2s can we take away from 8?

Children will use their knowledge of counting up in multiples to solve division calculations and recognise that this in the **inverse of multiplication**.

Children will continue to group and share equally using concrete equipment and will now begin to organise their groups into an array rather than scattered groupings.

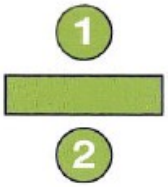


Stage 2 (cont.)

The direct link between multiplication and division should be made explicit when using models and representations.

Children will continue to make links between division and fractions. They will be aware that the division sign is the equivalent to the fraction line and so $p \div q$ can be written as $\frac{p}{q}$.

$1 \div 2$



Stage 3

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

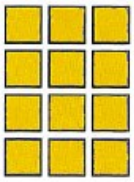
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

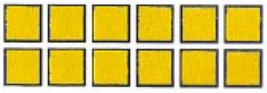
halve, share, share equally, one each, two each, three each..., divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____, ____ equal groups

Children will continue to use their knowledge of counting in multiples to support the **inverse of multiplication** and **repeated subtraction**.

Children will build on their use of concrete arrays for division recognising the links to **repeated subtraction** and the **inverse of multiplication** in order to derive the associated division facts. Children need to explore related division facts of a given number by making a variety of arrays and explaining what they show.

Representing 12

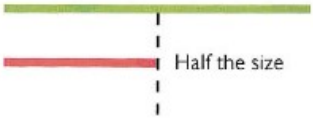




12 into ____ equal groups gives ____ in each group

12 into equal groups of ____ gives ____ groups

The children should be confident with their use of the language of scaling when talking about division with links made to simple fractions (e.g. half the size, three times smaller).



Stage 4

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total')
- Use the language '**digit**' not number (number is the amount or quantity)

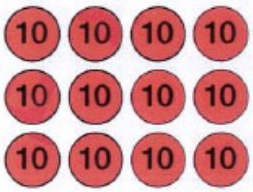
÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____ , ____ equal groups

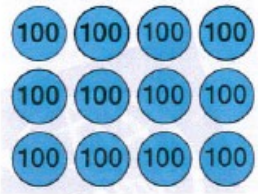
Children will continue to organise groups into an array now working with larger numbers by either grouping or sharing. Children will be able to explain all the facts they know about a given array with no remainder. They should be making arrays with the equipment to establish 'How many in each group?' or 'How many groups?'. Children should continue to experience the language of scaling (e.g. scaling down pictures by dividing by powers of 10, dividing by powers of 1000 in converting between units of measure)

120 ÷ 3



120 shared equally between 3 is 40.
120 shared equally between 4 is 30.
3 equal groups of 40 make 120.
4 equal groups of 30 make 120.

1200 ÷ 3



1200 shared equally between 3 is 400.
1200 shared equally between 4 is 300.
3 equal groups of 400 make 120.
4 equal groups of 300 make 1200.

Stage 5

POINTS TO REMEMBER

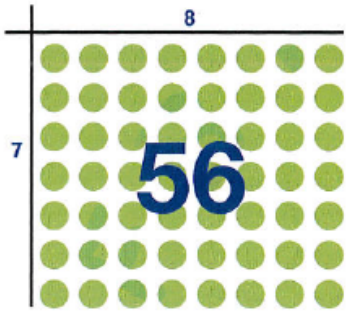
- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each..., divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____, ____ equal groups

Children will continue to work with concrete arrays, exploring known multiplication/division facts, with the use of grid lines to begin to make the link to short division where numbers are easily divisible. The children understand that the array within short division can be interpreted for both sharing between or equal groups of where the dots within the array each represent 1.



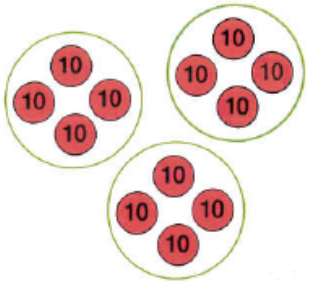
How many equal groups of 7 can I make?
(grouping is represented in the columns)

or

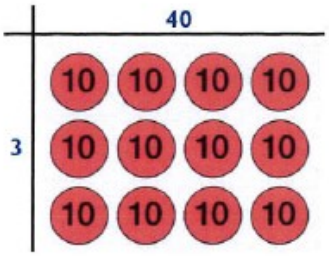
If I put these into 7 equal groups, how many in each group?
(sharing between is represented in the rows)

Children will begin to use counters within an array to show the sharing model of division, using their knowledge of the **principle of exchange** where necessary. At this stage, children are encouraged to consider the links between the sharing model and fractions.

120 can be exchanged for 12 tens in order to make an array



120 shared into 3 equal groups gives 40 in each group



We can explicitly see 40 three times; 3 rows of 40, a ⅓ of 120 is 40.
We can divide the array into three parts and there is 40 in each part.

Stage 6

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

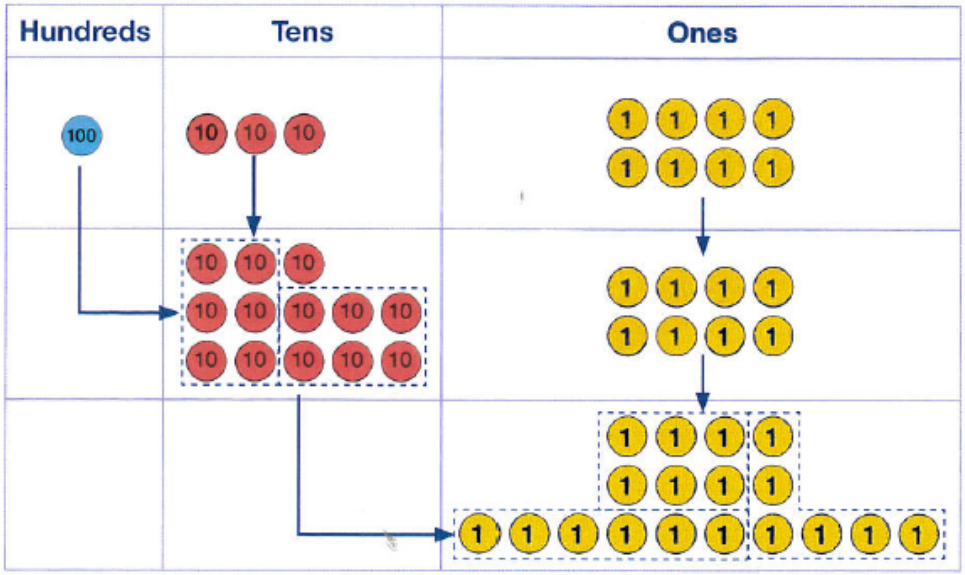
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ____ , ____ equal groups

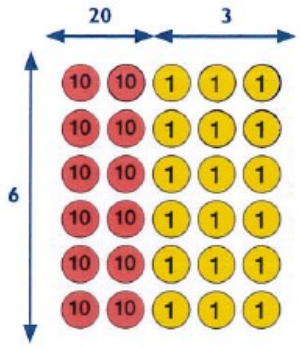
Children will work with equipment to divide any integer by a single digit divisor using their sound knowledge of the principle of exchange.

They will begin to be introduced to numbers that have remainders and will recognise and be able to talk about these when they do not 'fit' into the array.

Children will be introduced to the notation of short division, linking with the principle of exchange and how this relates to the practical representations.

Children continue to be encouraged to consider the links between the sharing model and fractions.





In the array, we can explicitly see 23 six times; 6 rows of 23. This is the sharing model.

⅙ of 138 is 23.

We can divide the array up into six equal parts and there is 23 in each part.

$$\begin{array}{r} 2\ 3 \\ 6\overline{)\ 1\ ^{1}3\ ^{1}8} \end{array}$$

Stage 7

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

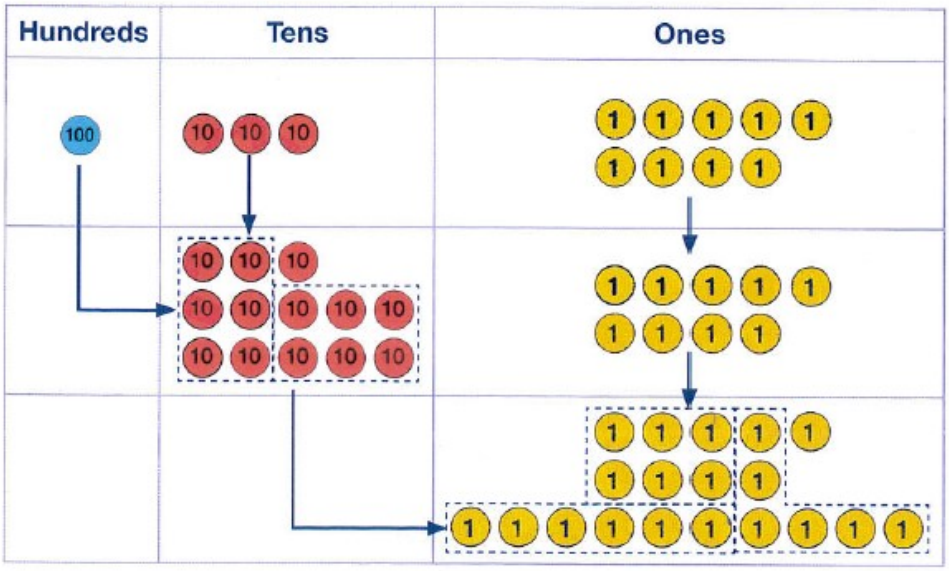
÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

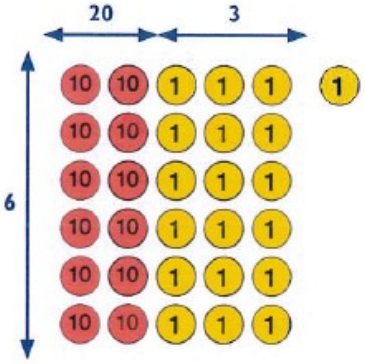
VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each..., divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___ equal groups

Children will now be secure in using short division for one-digit divisors with an integer quotient.

They will now begin to use the short division notation for calculations involving remainders.





$$\begin{array}{r} \phantom{6\overline{)}}\ 2\ \ 3\ \text{r}1 \\ 6\overline{)\ \not{1}\ {}^{1}3\ {}^{1}9} \end{array}$$

Children will also begin to explore the use of jottings of friendly numbers to support long division of calculations with 2-digit divisors.

1 x 15 = 15
2 x 15 = 30
4 x 15 = 60
8 x 15 = 120
10 x 15 = 150

20 x 15 = 300

$$\begin{array}{r} \phantom{15\overline{)}}\ 2\ \ 8 \\ 15\overline{)\ 4\ 2\ 0} \\ -\ 3\ 0\ 0 \quad (20 \times 15) \\ \hline \ 1\ 2\ 0 \\ -\ 1\ 2\ 0 \quad (8 \times 15) \\ \hline \ 0 \end{array}$$

Stage 8

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total')
- Use the language '**digit**' not number (number is the amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

halve, share, share equally, one each, two each, three each..., divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___ equal groups

Children will now be secure in using short division for one-digit divisors and long division for two-digit divisors with an integer quotient.

They will now explore the use of long division for two-digit divisors which may include a remainder.

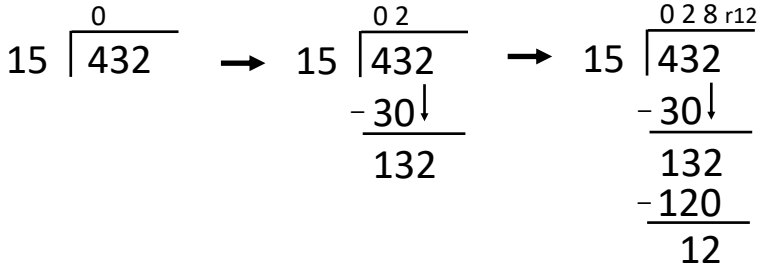
The children will begin to interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.

1 x 15 = 15
2 x 15 = 30
4 x 15 = 60
8 x 15 = 120
10 x 15 = 150

20 x 15 = 300

	2 8 r12	2 8 $\frac{12}{15}$	2 8 $\frac{4}{5}$
15 |	4 3 2	4 3 2	4 3 2
−	3 0 0 (20 x 15)	3 0 0 (20 x 15)	3 0 0 (20 x 15)
	1 3 2	1 3 2	1 3 2
−	1 2 0 (8 x 15)	1 2 0 (8 x 15)	1 2 0 (8 x 15)
	1 2	1 2	1 2

Once children are confident using this method for long division, they may then move on to the abbreviated layout (below) for long division. It is important that the above step is understood in terms of size/ place value of numbers as well as security with grouping before moving onto the reduced form.



Calculating with decimals

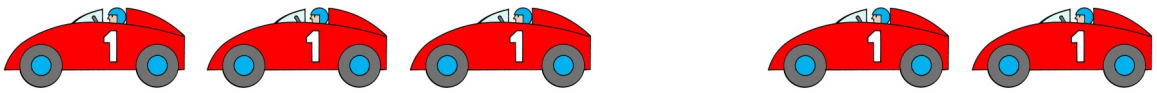
When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

SUPPLEMENTARY GUIDANCE

UNDERSTANDING IN MATHS

"The success of an activity is based on whether it represents the maths in a way the child understands." **J.Bruner**

1) **ENACTIVE**—internalised *action* with objects eg. moving 3 cars and 2 cars beside each other.

e.g. 

2) **ICONIC**—sensory imagery or pictures.

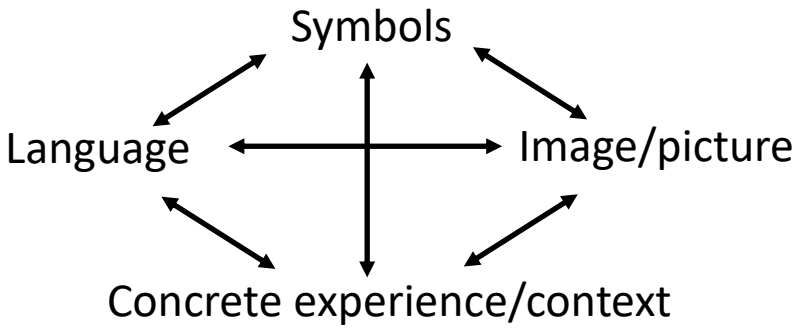
e.g. 

3) **SYMBOLIC**—arbitrary symbols, talk.

e.g. 'three plus two' or '3+2'

All learning in mathematics should transition through these three stages to secure understanding.

"The teacher's role in developing understanding is… to help the child build up connections between new experiences and previous learning." **Haylock & Cockburn**



"...Pupils should make connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems…" **National Curriculum**

"Schools should develop the expertise of staff: In choosing teaching approaches and activities that foster pupils' deeper understanding, I checking and probing pupils' understanding during the lesson, in understanding the progression in strands of maths over time, so that they know the key knowledge and skills that underpin each stage of learning…" **OFSTED**

PROMPTING CHILDREN'S THINKING THROUGH QUESTIONING

The following key questions, ideas and strategies should be used in maths lessons to deepen understanding and prompt thinking/reasoning skills:

What do you notice? What's the same? What's different?

- Spot the mistake/Which is correct?
- True or false?
- What comes next?
- Do, then explain
- Make up an example/Write more statements
- Possible answers/Other possibilities
- Continue the pattern
- Missing numbers/symbols/information
- Working backwards/Use the inverse/ Undoing
- Hard and easy questions
- What else do you know?/Use a fact
- Fact families
- Convince me/Prove it/Generalising
- Make an estimate/Size of an answer
- Always, sometimes, never
- Making links/Application
- Can you find?
- Odd one out
- Complete the pattern/Continue the pattern
- Another and another
- Ordering
- Testing conditions
- The answer is…
- Visualising

NCETM 2015
Reasoning Strategies/ Pedagogies

It is important to ensure that there are lots of opportunities for maths talk to happen throughout the lesson.

As well as the above questions try to include the following key ideas:

- Enabling learning through:
  - > **Drawing attention to...**
  - **> Developing reasoning and making connections.**
- Providing opportunities for children to:
  - **> Manipulate, experience, see**
  - **> Engage in talk (listen, analyse and discuss)**
- Developing children's thinking through:
  - **> Investigation**
  - **> Scaffolding**

Opportunities in maths learning should include:

- **MATHEMATICAL THINKING—**reasoning to apply understanding and skills to solve problems.
- **PROPORTIONALITY–** understanding size and relationships of one thing to another.
- **PATTERN–** the structure and relationships in mathematical concepts.
- **GENERALITY–** connecting structures and relationships to apply rules.
- **REPRESENTATION–** representing questions in a variety of ways (enactive/ iconic/symbolic)

PLACE VALUE

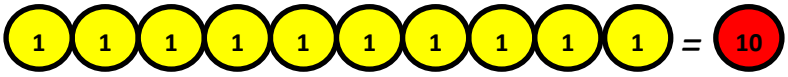
The New Curriculum has an increased emphasis on place value understanding and mastery.

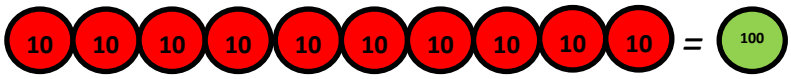
- All numbers can be represented using the ten digits 0,1,2,3,4,5,6,7,8,9
- The value that each digit represents in determined by its place
- The principle of exchange—that when you have accumulated then in one place, it can be exchanged for one in the next place to the left.
- Connect how we say words and how we write them. (Nb. 'teen numbers)
- Understand how 0 is used as a place holder.
- Connect the symbols for numbers with the numberline.

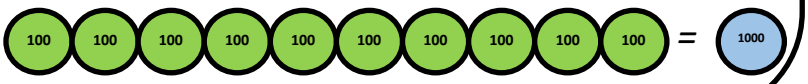
EXCHANGE

'ten' 10 is the most important in our naming system because when we are counting collections, as soon as we have a group of ten, we call them something else.

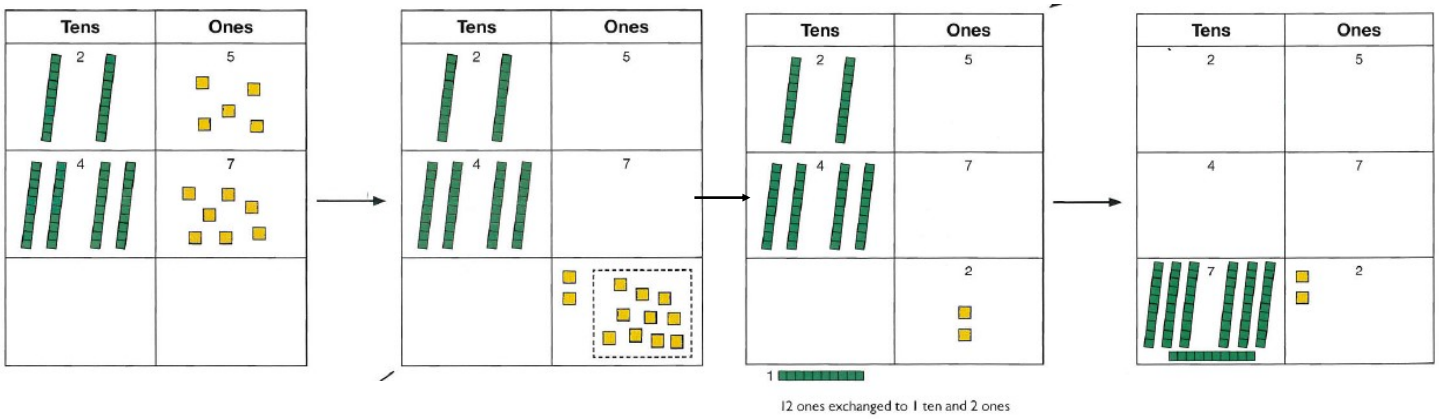
So:

ten *ones* are called one *ten* 

ten *tens* are called one *hundred* 

ten *hundreds* are called one *thousand* 

25 + 47



12 ones exchanged to 1 ten and 2 ones

PLEASE NOTE—We now need to use 'ONES' as the place value column name. Eg. HUNDREDS/TENS/ONES

The word UNITS should be phased out due to misinterpretation of language.

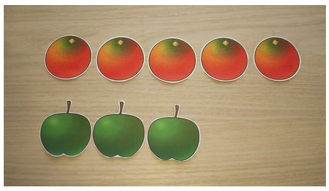
SINGAPORE BAR METHOD

http://www.bbc.co.uk/skillswise/0/24925787

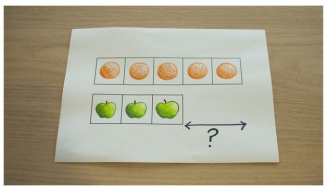
'Singapore' maths has a strong emphasis on handling concrete things moving on to drawing one-to-one pictorial iconic representations of them, to eventually understanding and using abstract symbols with confidence. Much of this system works around the bar method.

STAGES OF THE BAR METHOD:

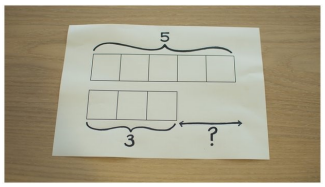
1) Imagine you have five oranges and three apples, how many more oranges than apples? (Concrete with actual objects)



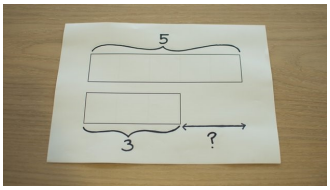
2) At first children model the problem with physical objects they can move around: like these cut-out pictures.



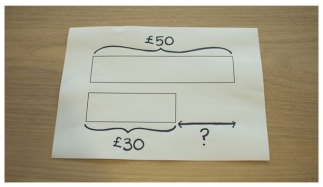
3) After a few months they start to draw pictures of the problem to help them think about it.



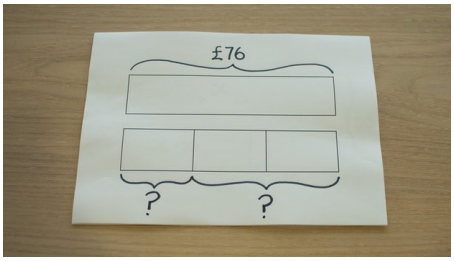
4) Over time children drop the pictures and just draw boxes. Then they start adding numbers as labels.



5) Once children are confident with the meaning of the number symbol they no longer need to draw all the boxes. However they know they can always draw the boxes in again if they need to convince themselves.

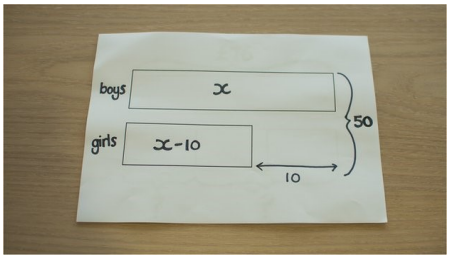


6) How much change if you pay for a £30 shirt with a £50 note? The model can be used to help visualise almost any maths problem.



7) Three people want to split a restaurant bill of £76. How much for a couple who want to pay together? The model helps break the problem down. First divide £76 by 3. Then times the answer by 2.

8) In a year group there are 50 children. There are 10 fewer girls than boys. How many boys? The model can help visualise the unknown quantity. You can see that x + x - 10 = 50. If you add the 10 you get x + x = 60. So x = 30



TIMES TABLES

https://www.youtube.com/watch?v=yXdHGBfoqfw

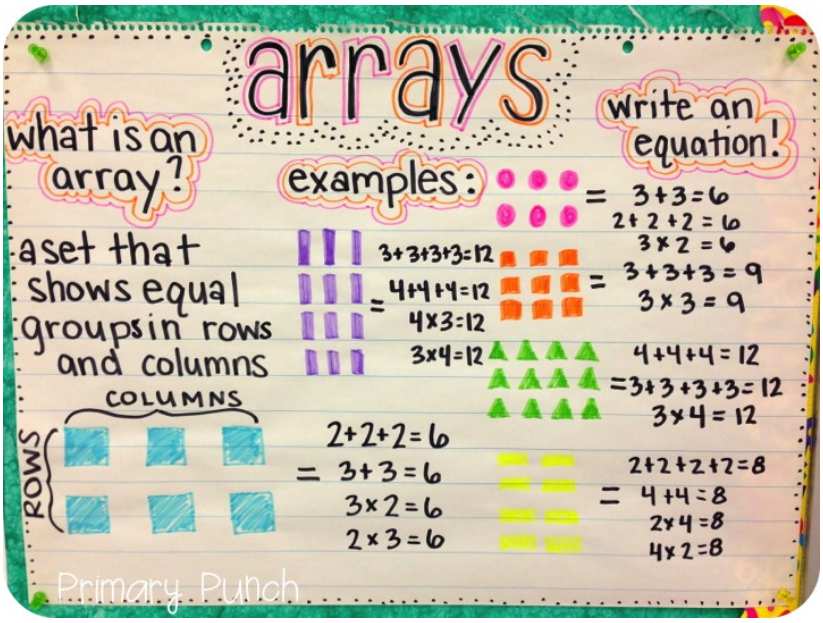
There is an increased emphasis on knowledge and recall of times-tables, with all children required to know up to 12x12 by the end of year 4.
Thinking back to the three stages of learning in maths—enactive, iconic and symbolic; are we presenting times-tables in these different ways to support learning/understanding rather than just recall?

The numberline technique used in the video above is working with the symbolic numbers but also supports this with the iconic representation of the number line via the counting stick.

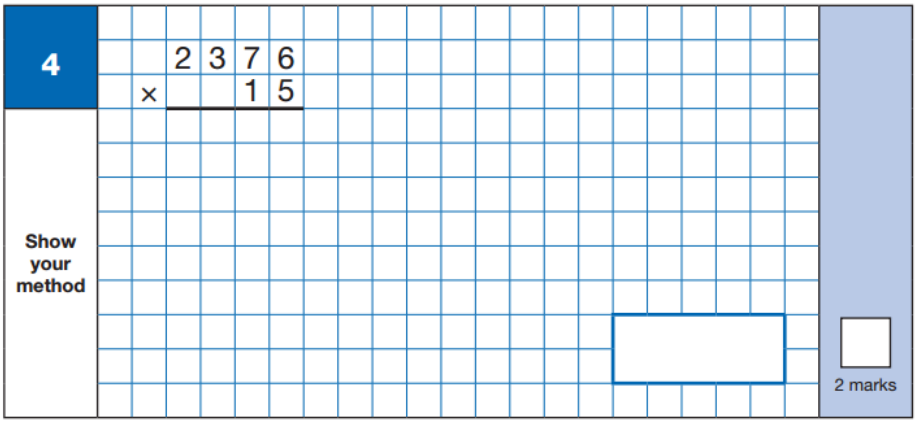
Times tables practice should ensure that children have moved through these three stages of learning, rather than just chanting alone. When vocalising times tables, children should be able to say them in different ways eg. Listing '3,6,9,12' as well as full '1 times 3 is three, 2 times three is 6'. Also encourage different language use eg. '1 multiplied by 3' or 'the product of 1 and 3'.

REMEMBER TO USE ARRAYS

Arrays are one of the crucial steps from enactive to iconic in times tables—they allow the children to make important links in maths, for example seeing multiplications as repeated addition and division as a direct inverse of multiplication (eg. Sharing out the chocolates from the box!)



SAMPLE Y6 SATS MATHS QUESTION



Qu	Requirement	Mark	Additional guidance
4	Award **TWO** marks for the correct answer of 35640 If the answer is incorrect award **ONE** mark for evidence of using the formal method of long multiplication which contains no more than one arithmetical error, eg: ▪ 2376 × 15 11880 23760 wrong answer	**Up to 2 marks**	Working must be carried through to reach an answer for the award of **ONE** mark. In all cases accept follow-through of **ONE** error in working. **Do not** award any marks if: ▪ The error is in the place value, eg by omission of the zero when multiplying by tens eg: 2376 × 15 11880 2376 wrong answer ▪ The final (answer) line of digits is missing

To achieve working out marks, the formal method has to be used.

Commentary: This question illustrates the increased demand of multiplying a 4-digit number by a 2-digit number. It is presented vertically, to encourage pupils to use a formal written method.
Only the use of a formal written method will gain the method mark should the pupil calculate incorrectly.

By the end of Year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Y6 STATUTORY REQUIREMENTS:

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the efficient written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Y4 STATUTORY REQUIREMENTS:

- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- add and subtract numbers with up to 4 digits using the efficient written methods of columnar addition and subtraction where appropriate

Y4 GUIDANCE:

Pupils should practise to become fluent in the efficient written method of short multiplication for multiplying using multi-digit numbers, and short division with exact answers when dividing by a one-digit number.

Y5 STATUTORY REQUIREMENTS:

- multiply numbers up to 4 digits by a one- or two-digit number using an efficient written method, including long multiplication for two-digit numbers
- divide numbers up to 4 digits by a one-digit number using the efficient written method of short division and interpret remainders appropriately for the context
- add and subtract whole numbers with more than 4 digits, including using efficient written methods (columnar addition and subtraction)

Y3 STATUTORY REQUIREMENTS:

- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to efficient written methods
- add and subtract numbers with up to three digits, using the efficient written methods of columnar addition and subtraction

Y2 GUIDANCE:

Recording addition and subtraction in columns supports place value and prepares for efficient written methods with larger numbers.