ST. HUGH'S CATHOLIC PRIMARY VOLUNTARY ACADEMY

MATHEMATICS CALCULATION POLICY

We develop children’s fluency with basic number facts

Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. A short time everyday should be spent on these basic facts and quickly leads to improved fluency (see NCETM mastering number). This can be done using simple whole class chorus chanting. The evidence suggests that this is not meaningless rote learning; rather, this is an important step to developing conceptual understanding through identifying patterns and relationships between the tables (for example, that the products in the 6× table are double the products in the 3× table). This helps children develop a strong sense of number relationships, an important prerequisite for procedural fluency.

A suggested order for learning x-tables is as follows to provide opportunities to make connections:

X 1	X 10	X5	X2	x4	x8	x3	x6	x9	x7	x11	x12

When children move onto their 11 and 12 x-tables, children are encouraged to make mathematical connections to see 11 as 10+1 and 12 as 10+2 in order to see the links between times-tables in combination. White Rose maths also has discrete work on multiplication tables (and related division facts) as children move up to Year 4 in preparation for their multiplication tables check.

We develop children’s fluency in mental calculation

Efficiency in calculation requires having a variety of mental strategies. It is crucial to emphasise the importance of 10 and partitioning numbers to bridge through 10. For example:

$$9 + 6 = 9 + 1 + 5 = 10 + 5 = 15.$$

It is helpful to make a 10 as this makes the calculation easier.

We develop fluency in the use of formal written methods

Teaching column methods for calculation provides the opportunity to develop both procedural and conceptual fluency. Children can understand the structure of the mathematics presented in algorithms, with a particular emphasis on place value. Base ten apparatus should be used and illustrated in books to support the development of fluency and understanding.

Informal methods of recording calculations are an important stage to help children develop fluency with formal methods of recording. They are stepping stones to formal written methods.

We develop children’s understanding of the = symbol

The symbol **=** is an assertion of equivalence. If we write:

$$3 + 4 = 6 + 1$$

then we are saying that what is on the left of the = symbol is necessarily equivalent to what is on the right of the symbol. But many children interpret = as being simply an instruction to evaluate a calculation, as a result of always seeing it used thus:

$$3 + 4 =$$

$$5 \times 7 =$$

$$16 - 9 =$$

If children only think of = as meaning “work out the answer to this calculation” then they are likely to get confused by empty box questions such as:

$$3 + \square = 8$$

Later they are very likely to struggle with even simple algebraic equations, such as:

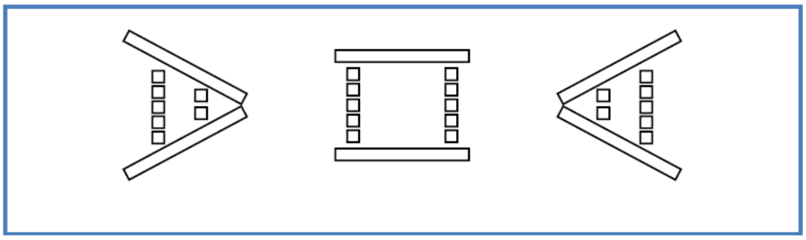
$$3y = 18$$

One way to model equivalence such as 2 + 3 = 5 is to use balance scales.

We should aim to vary the position of the = symbol and include empty box problems to deepen children's understanding of the = symbol.

Teach inequality alongside teaching equality

To help young children develop their understanding of equality, they also need to develop understanding of inequality. One way to introduce the < and > signs is to use rods and cubes to make a concrete and visual representations such as:



to show that 5 is greater than 2 (5 > 2), 5 is equal to 5 (5 = 5), and 2 is less than 5 (2 < 5).

Balance scales can also be used to represent inequality.

Incorporating both equality and inequality into examples and exercises can help children develop their conceptual understanding. For example, in this empty box problem children have to decide whether the missing symbol is <, = or >:

$$5 + 7 \square 5 + 6$$

An activity like this also encourages children to develop their mathematical reasoning: "I know that 7 is greater than 6, so 5 plus 7 must be greater than 5 plus 6".

Asking children to decide if number sentences are true or false also helps develop mathematical reasoning. For example, in discussing this statement:

$$4 + 6 + 8 > 3 + 7 + 9$$

a child might reason that "4 plus 6 and 3 plus 7 are both 10. But 8 is less than 9. Therefore 4 + 6 + 8 must be less than 3 + 7 + 9, not more than 3 + 7 + 9".

In both these examples the numbers have been deliberately chosen to allow the children to establish the answer without actually needing to do the computation. This emphasises further the importance of mathematical reasoning.

We have an emphasis on calculation

Young children benefit from being helped at an early stage to start calculating, rather than relying on 'counting on' as a way of calculating. For example, with a sum such as:

$$4 + 7 =$$

Rather than starting at 4 and counting on 7, children could use their knowledge and bridge to 10 to deduce that because 4 + 6 = 10, so 4 + 7 must equal 11. This is supported in school by use of the Mastering Number Programme from Foundation Stage through to Year 2.

Look for pattern and make connections

We aim to use a great many visual representations of the mathematics and some concrete re-sources. Understanding, however, does not happen automatically, children need to reason by and with themselves and make their own connections. Throughout the school, children will be encour-aged to look for pattern and connections in the mathematics. The question "What's the same, what's different?" is used frequently to make comparisons. For example *"What's the same, what's different between the three times table and the six times table?"* Visual representations and models/images are used frequently as part of White Rose maths scheme throughout the school both for lesson delivery and activities.

We use intelligent practice

In designing exercises for children, teachers ensure an appropriate path for practicing the thinking process with increasing creativity (Gu, 1991) via the White Rose schemes of learning. This will allow children to develop both procedural and conceptual fluency. Children are required to reason and make connections between calculations. The connections made improve their fluency. The example calculations below show how linked calculations can help to develop children's understanding:

2 × 3 =	6 × 7 =	9 × 8 =
2 × 30 =	6 × 70 =	9 × 80 =
2 × 300 =	6 × 700 =	9 × 800 =
20 × 3 =	60 × 7 =	90 × 8 =
200 × 3 =	600 × 7 =	900 × 8 =

We use empty box problems

Empty box problems are a powerful way to help children develop a strong sense of number through intelligent practice. They provide the opportunity for reasoning and finding easy ways to calculate. They enable children to practise procedures, whilst at the same time thinking about conceptual connections.

A sequence of examples such as

$$3 + \square = 8$$

$$3 + \square = 9$$

$$3 + \square = 10$$

3 + □ = 11

helps children develop their understanding that the = symbol is an assertion of equivalence, and invites children to spot the pattern and use this to work out the answers.

This sequence of examples does the same at a deeper level:

3 × □ + 2 = 20

3 × □ + 2 = 23

3 × □ + 2 = 26

3 × □ + 2 = 29

3 × □ + 2 = 35

Children should also be given examples where the empty box represents the operation, for example:

4 × 5 = 10 □ 10

6 □ 5 = 15 + 15

6 □ 5 = 20 □ 10

8 □ 5 = 20 □ 20

8 □ 5 = 60 □ 20

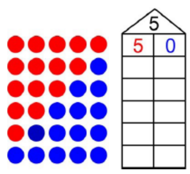
These examples also illustrate the careful use of variation to help children develop both procedural and conceptual fluency.

We expose mathematical structure and work systematically

Developing instant recall alongside conceptual understanding of number bonds to 10 is important.

The Mastering Number Programme and Number Sense Maths are both used in school to support this conceptual understanding.

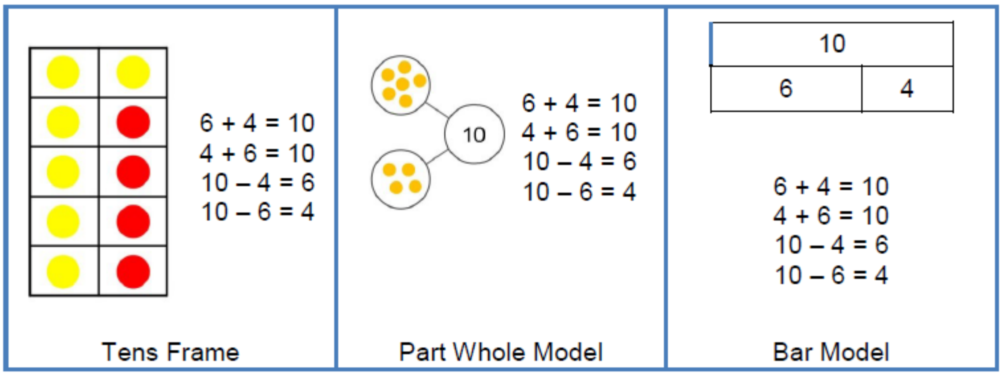
This can be supported through the use of images such as the example illustrated below:



Shanghai Textbook Grade 1 (aged 6/7)

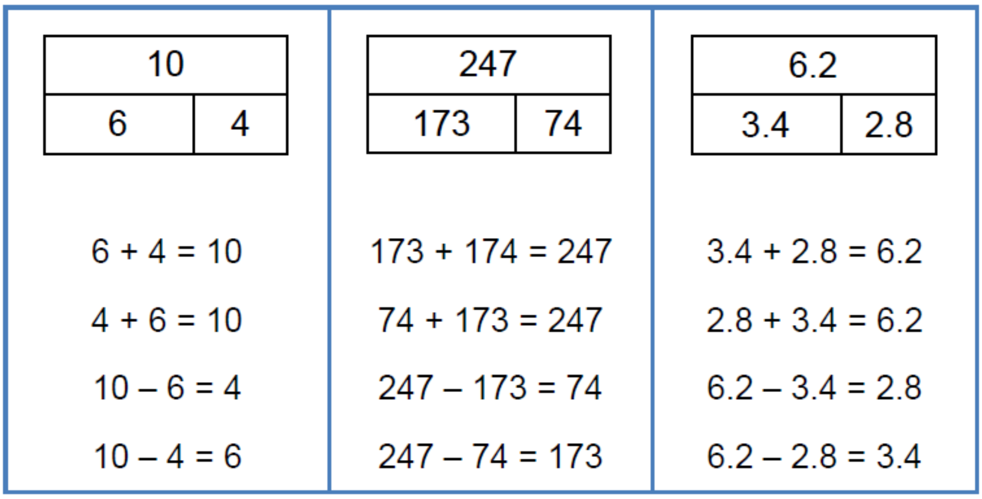
The image lends itself to seeing pattern and working systematically and children can connect one number fact to another and be certain when they have found all the bonds to 5.

Using other structured models such as tens frames, part whole models or bar models can help children to reason about mathematical relationships.



Connections between these models should be made, so that children understand the same mathematics is represented in different ways. Asking the question "What's the same what's different?" has the potential for children to draw out the connections.

Illustrating that the same structure can be applied to any numbers helps children to generalise mathematical ideas and build from the simple to more complex numbers, recognising that the structure stays the same; it is only the numbers that change. For example:



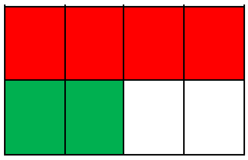
We move between the concrete and the abstract

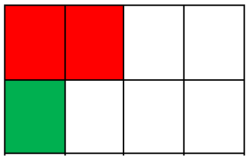
Children's conceptual understanding and fluency is strengthened if they experience concrete, visual and abstract representations of a concept during a lesson. Moving between the concrete and the abstract helps children to connect abstract symbols with familiar contexts, thus providing the opportunity to make sense of, and develop fluency in the use of, abstract symbols.

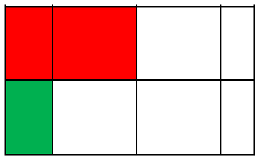
For example, in a lesson about addition of fractions children could be asked to draw or complete a picture to represent the sum

$$\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

Alternatively, or in a subsequent lesson, they could be asked to discuss which of three visual images correctly represents the sum, and to explain their reasoning:







We contextualise the mathematics

A lesson about addition and subtraction could start with this contextual story:

"There are 11 people on a bus. At the next stop 4 people get on. At the next stop 6 people get off. How many are now on the bus?"

This helps children develop their understanding of the concepts of addition and subtraction. But during the lesson the teacher should keep returning to the story. For example, if the children are thinking about this calculation

14 – 8

then the teacher should ask the children:

"What does the 14 mean? What does the 8 mean?", expecting that children will answer:

"There were 14 people on the bus, and 8 is the number who got off."

Then asking the children to interpret the meaning of the terms in a sum such as 7 + 7 = 14 will give a good assessment of the depth of their conceptual understanding and their ability to link the concrete and abstract representations of mathematics.

We use questioning to develop mathematical reasoning

Teachers' questions in mathematics lessons are often asked in order to find out whether children can give the right answer to a calculation or a problem. But in order to develop children's conceptual understanding and fluency there needs to be a strong and consistent focus on questioning that encourages and develops their mathematical reasoning.

This can be done simply by asking children to explain how they worked out a calculation or solved a problem, and to compare and contrast different methods that are described. Children should quickly come to expect that they need to explain and justify their mathematical reasoning, and they should soon start to do so automatically – and enthusiastically. Some calculation strategies are more efficient; teachers should scaffold children's thinking to guide them to the most efficient methods, whilst at the same time valuing their own ideas.

Rich questioning strategies include:

- *"What's the same, what's different?"*

In this sequence of expressions, what stays the same each time and what's different?

23 + 10	23 + 20	23 + 30	23 + 40

Discussion of the variation in these examples can help children to identify the relationship between the calculations and hence to use the pattern to calculate the answers.

- *"Odd one out"*

Which is the odd one out in this list of numbers: 24, 15, 16 and 22?

This encourages children to apply their existing conceptual understanding.

Possible answers could be:

"15 is the odd one out because it's the only odd number in the list."

"16 is the odd one out because it's the only square number in the list."

"22 is the odd one out because it's the only number in the list with exactly four factors."

If children are asked to identify an 'odd one out' in this list of products:

24 × 3 36 × 4 13 × 5 32 × 2

they might suggest:

"36 × 4 is the only product whose answer is greater than 100."

"13 × 5 is the only product whose answer is an odd number."

- *"Here's the answer. What could the question have been?"*

Children are asked to suggest possible questions that have a given answer. For example, in a lesson about addition of fractions, children could be asked to suggest possible ways to complete this sum:

$$\square + \square = \frac{3}{4}$$

- *Identify the correct question*

Here children are required to select the correct question:

A 3.5m plank of wood weighs 4.2 kg

The calculation was:

3.5 ÷ 4.2

Was the question:

a. How heavy is 1m of wood?

b. How long is 1kg of wood?

- *True or False*

Children are given a series of equations are asked whether they are true or false:

4 × 6 = 23 4 × 6 = 6 × 4 12 ÷ 2 = 24 ÷ 4 12 × 2 = 24 × 4

Children are expected to reason about the relationships within the calculations rather than calculate.

• *Greater than, less than or equal to >, <, or =*

$3.4 \times 1.2 \bigcirc\ 3.4$ $5.76 \bigcirc 5.76 \div 0.4$ $4.69 \times 0.1 \bigcirc 4.69 \div 10$

These types of questions are further examples of intelligent practice where conceptual understanding is developed alongside the development of procedural fluency. They also give pupils who are, to use Ofsted's phrase, *rapid graspers* the opportunity to apply their understanding in more complex ways.

We expect children to use correct mathematical terminology and to express their reasoning in complete sentences

The quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to use correct mathematical terminology and to explain their mathematical thinking in complete sentences.

I say, you say, you say, you say, we all say

This example technique enables the teacher to provide a sentence stem for children to communicate their ideas with mathematical precision and clarity. These sentence structures often express key conceptual ideas or generalities and provide a framework to embed conceptual knowledge and build understanding. As part of the White Rose scheme as well as the NCETM Mastering Number programme and Number Sense Maths, stem sentences are frequently used to support children's correct use of mathematical vocabulary.

For example:

If the rectangle is the whole, the shaded part is one third of the whole.

Having modelled the sentence, the teacher then asks individual children to repeat this, before asking the whole class to chorus chant the sentence. This provides children with a valuable sentence for talking about fractions. Repeated use helps to embed key conceptual knowledge.

Another example is where children fill in the missing parts of a sentence; varying the parts but keeping the sentence stem the same. For example:

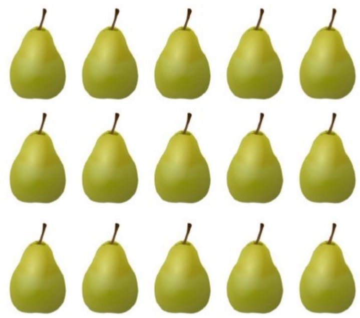
There are 12 **stars**. $\frac{1}{3}$ of the **stars** is equal to **4 stars**

Children use the same sentence stem to express other relationships. For example:

There are 12 **stars**. $\frac{1}{4}$ of the **stars** is equal to **3 stars**

There are 12 **stars**. $\frac{1}{2}$ of the **stars** is equal to **6 stars**

Similarly:



There are **15 pears**. $\frac{1}{3}$ of the **pears** is equal to **5 pears**

There are **15 pears**. $\frac{1}{5}$ of the **pears** is equal to **3 pears**

When talking about fractions it is important to make reference to the whole and the part of the whole in the same sentence. The above examples help children to get into the habit of doing so.

Another example is where a mathematical generalisation or "*rule"* emerges within a lesson. For example:

When adding 10 to a number, the ones digit stays the same

This is repeated in chorus using the same sentence, which helps to embed the concept.

We identify difficult points

Difficult points need to be identified and anticipated when lessons are being designed and these need to be an explicit part of the teaching, rather than the teacher just responding to children's difficulties if they happen to arise in the lesson. The teacher should be actively seeking to uncover possible difficulties because if one child has a difficulty it is likely that others will have a similar difficulty. Difficult points also give an opportunity to reinforce that we learn most by working on and through ideas with which we are not fully secure or confident. Discussion about difficult points can be stimulated by asking children to share thoughts about their own examples when these show errors arising from insufficient understanding. For example:

$$\frac{2}{14} - \frac{1}{7} = \frac{1}{7}$$

MATHEMATICAL VOCABULARY AND SPELLING

KS1

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at Key Stage 1.

YEAR 3/4

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

YEAR 5/6

Pupils should read, spell and pronounce mathematical vocabulary correctly.

Glossary of terms for teachers

Cardinal number	The number of items in a set, the quantity but not the order of things. For example, 'There are five pencils in a pot.'
Conservation of number	If a group of objects is rearranged, the total number of objects stays the same.
Consecutive	Following in order. Consecutive numbers are adjacent in a count. For example, 5, 6, 7 are consecutive numbers. 25, 30, 35 are consecutive multiples of 5.
Commutativity	For addition and multiplication, the numbers in a calculation can be done in any order and will result in the same answer. E.g. 3 x 4 = 12 and 4 x 3 = 12 or 3 + 4 = 7 and 4 + 3 = 7. Addition and multiplication and communicative. Subtraction and Division are not communicative. However, children must understand that the numbers in a calculation can also be in any order but will result in a different answer. E.g. 7 – 5 = 2 and 5 – 7 =-2.
Digit	One of the symbols of a number system, most commonly the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. For example, the number 29 is a two-digit number;5 is a one-digit number. The position or place of a digit in a number conveys its value.
Dividend	The quantity which is to be divided. E.g. in the calculation, 12 ÷ 3, the dividend is 12.
Divisor	The quantity by which another quantity is to be divided. E.g. for the calculation, 12 ÷ 3, the divisor is 3.
Estimate	**Verb:** To arrive at a rough or approximate answer **Noun:** A rough or approximate answer
Fewer	Used to compare two or more sets of countable (discrete) objects. For example, 'There are fewer apples in this bag.'
Less	Used to compare 'uncountable' (continuous) quantities including measures. For example, 'This bottle has less water in it than that one'.
Long Multiplication	A formal calculation strategy that builds on understanding of the grid method into a compact column method. The multiplier is larger than 12 and therefore is partitioned during the process to aid calculation. Long multiplication is a multi-stage calculation which requires a final addition calculation in order to reach the final outcome.
Inverse of multiplication (as a method of division)	Counting up from 0 in multiples to reach a number in order to solve a division calculation. Inverse of multiplication is used to see how many amounts make a given number. E.g. starting at 0 and counting up in steps of 3 until 12 is reached. Some children find counting on in the multiples from 0 easier that repeated subtraction and this is fine so long as they understand they are using the inverse of multiplication rather than repeated subtraction
Number line	A line on which numbers are represented by points. Division marks are numbers, rather than spaces. They begin at any number and extend into negative numbers. They can show any number sequence. 012345678910
Number track	A numbered track along which counters may be moved. The number in a region represents the number of single moves from the start. Each number occupies a cell and is used to number the cell Numbers may have a matching illustration Supports learning to read numbers in numerals Supports locating ordered numbers They should start at 1 and not 0.
Numeral	A symbol used to denote a number. For example, 5, 23 and the Roman V are all numbers written in numerals.
Ordinal numbers	A term that describes a position within an ordered set. For example, first, second, third, fourth…twentieth.
Partition	To separate a set into subsets To split a number into component parts. For example, the two-digit number 38 can be partitioned into 30 + 8 or 19 + 19.

Pattern	A systematic arrangement of numbers, shapes or other elements according to a rule.
Principle of Exchange	The naming system when counting collections, that as soon as we have a group of ten we call them something else. The number we call ten (10 in numerals) is the most important in our naming system. E.g. ten ones are called one ten, ten tens are called one hundred; ten hundreds are called one thousand.
Proportionally	The relationship of one thing to another in terms of quantity, size, or number/out of the whole/2 out of 5. Proportionally puts the emphasis on the relationship rather than the quantity
Quotient	The result of a division calculation. E.g. In the calculation of 12 ÷ 3, the quotient is 4.
Ratio	The comparison of two properties /2:3. All ratio relationships are proportional
Repeated subtraction	Repeatedly subtracting the same amount each time in order to solve a division calculation. The idea of repeated subtraction should be ‘how many times can I take ____ away from ____? E.g. 12 ÷ 3 using repeated subtraction we should start as 12 and repeatedly count down in steps of 3 until 0 is reached.
Representation	The wide variety of ways to capture an abstract mathematical concept or relationship. This may be visible, such as a number sentence, a display of manipulative materials, or a graph, but it may also be an internal way of seeing and thinking about mathematical idea. Regardless of their form, representations can enhance students’ communication, reasoning, and problem-solving abilities; help them make connections among ideas; and aid them in learning new concepts and procedures.
Sequence	An ordered set of numbers or shapes arranged according to a rule
Short Multiplication	A formal calculation strategy that builds on understanding of the grid method into a compact column method. The multiplier is 12 or less and therefore is not partitioned during the process as the calculations should rely on knowledge of key multiplication facts up to 12 x 12. An expanded short multiplication method details each stage in brackets and shows clear connections to the grid method. This will be used as a vital stage in bridging the understanding from the grid method to short multiplication.
Subitising	This is the process whereby we recognise the size of a set, its cardinality, from the pattern or structure without having to count the number of objects. For example, recognising there are five dots in this pattern.
Zero	Nought or nothing In a place-value system, a place-holder. For example, 105. The cardinal number of an empty set.

Year 1 - 6

Calculation Policy

Addition and Subtraction

#MathsEveryoneCan

White Rose Maths

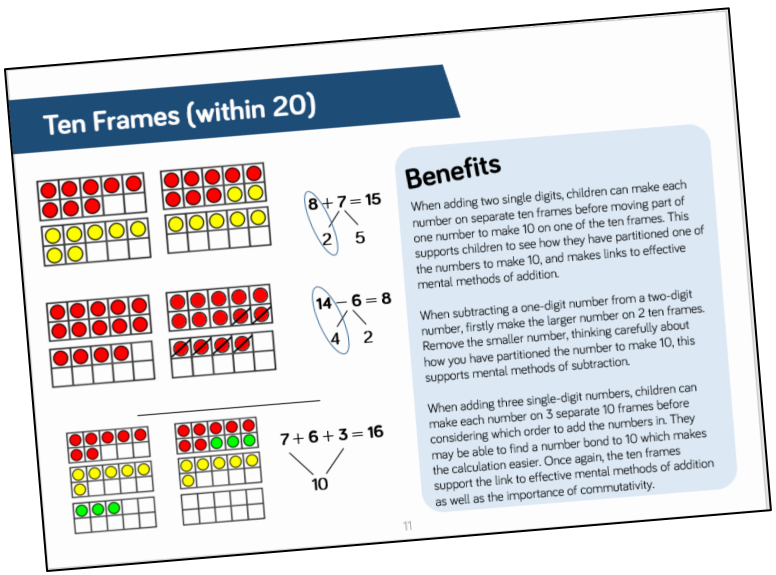
Notes and Guidance

Calculation Policy

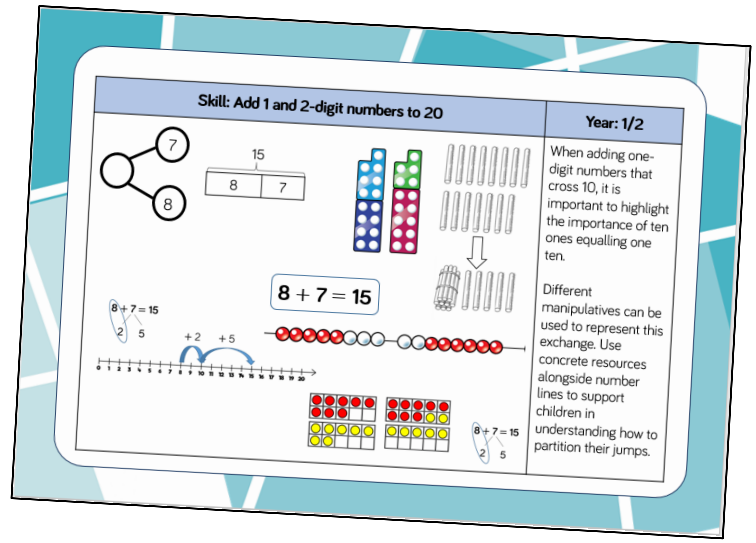
Welcome to the White Rose Maths Calculation Policy.

This document is broken down into addition and subtraction, and multiplication and division.

At the start of each policy, there is an overview of the different models and images that can support the teaching of different concepts. These provide explanations of the benefits of using the models and show the links between different operations.

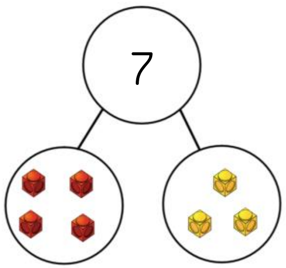


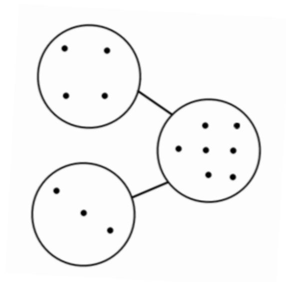
Each operation is then broken down into skills and each skill has a dedicated page showing the different models and images that could be used to effectively teach that concept.

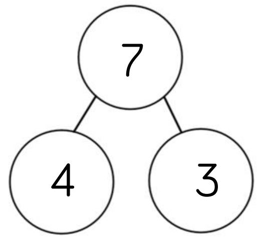


There is an overview of skills linked to year groups to support consistency through out school. A glossary of terms is provided at the end of the calculation policy to support understanding of the key language used to teach the four operations.

Part-Whole Model

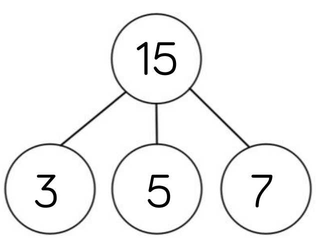


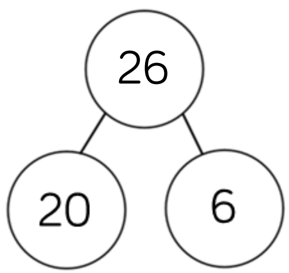


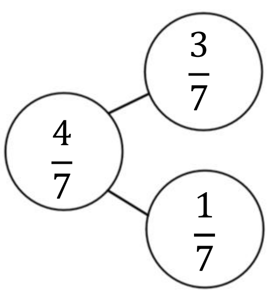


$$7 = 4 + 3$$
$$7 = 3 + 4$$

$$7 - 3 = 4$$
$$7 - 4 = 3$$







Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

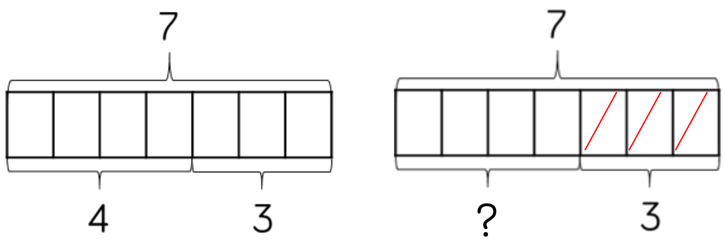
Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

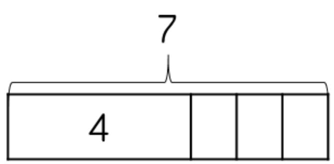
Bar Model (single)

Concrete

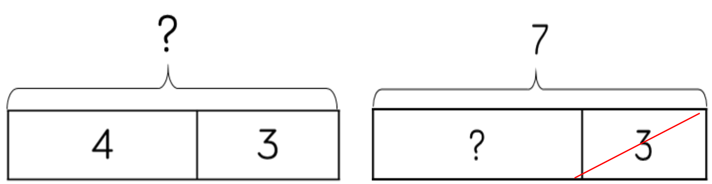
Discrete

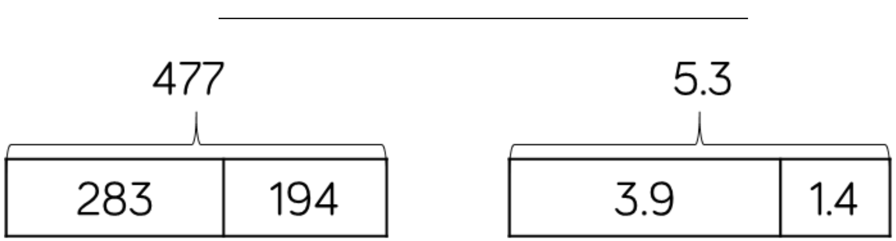


Combination



Continuous





Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

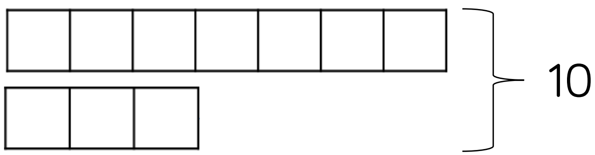
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

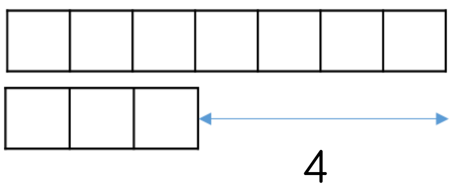
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

Discrete

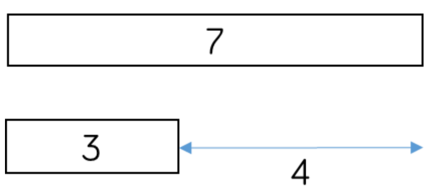


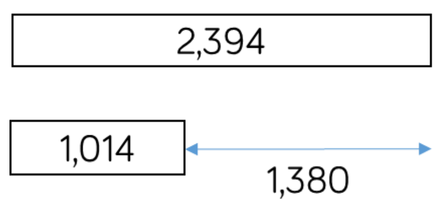
$$7 + 3 = 10$$



$$7 - 3 = 4$$

Continuous





$$7 - 3 = 4$$

$$2{,}394 - 1{,}014 = 1{,}380$$

Benefits

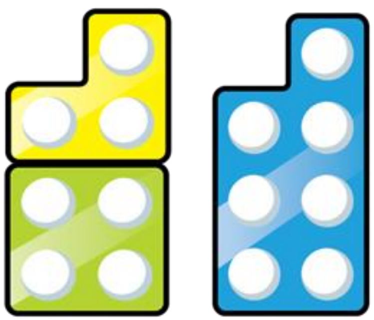
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

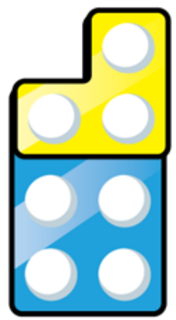
Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

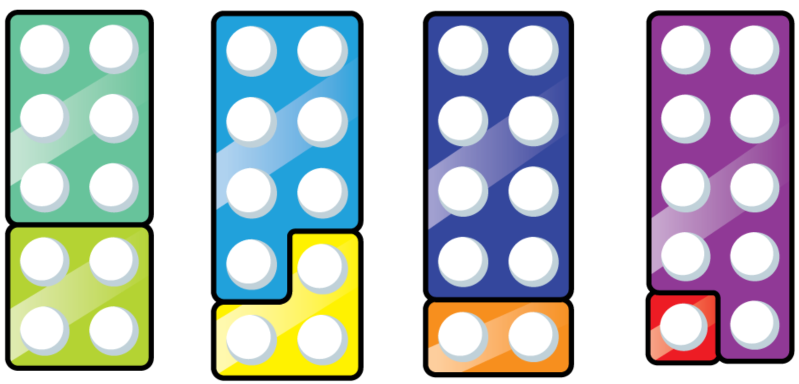
Number Shapes



7 = 4 + 3   7 = 3 + 4



7 − 3 = 4



6+4   7+3   8+2   9+1

Benefits

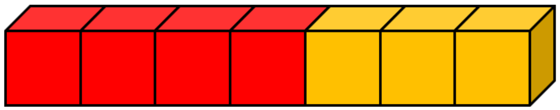
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

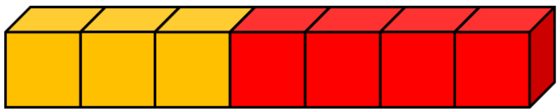
When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

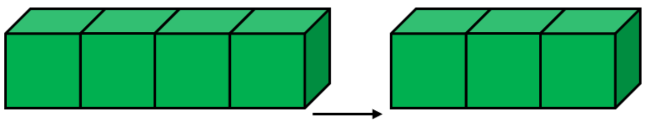
Cubes



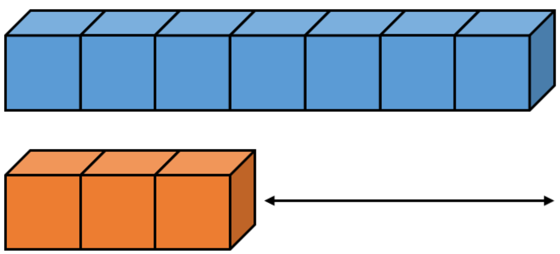
$7 = 4 + 3$



$7 = 3 + 4$



$7 - 3 = 4$



$7 - 3 = 4$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

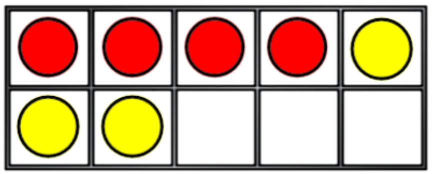
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

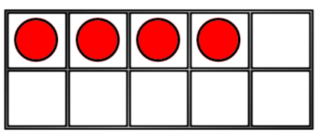
Ten Frames (within 10)

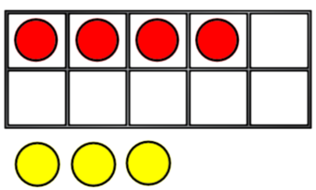


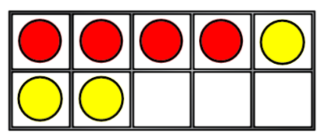
$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.

First | Then | Now

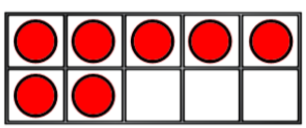


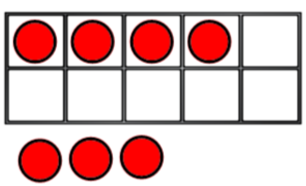


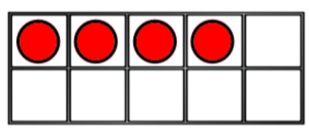


$4 + 3 = 7$

First | Then | Now







$7 - 3 = 4$

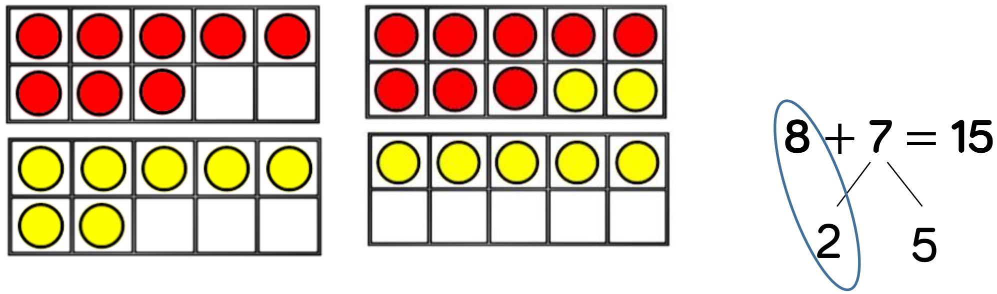
Benefits

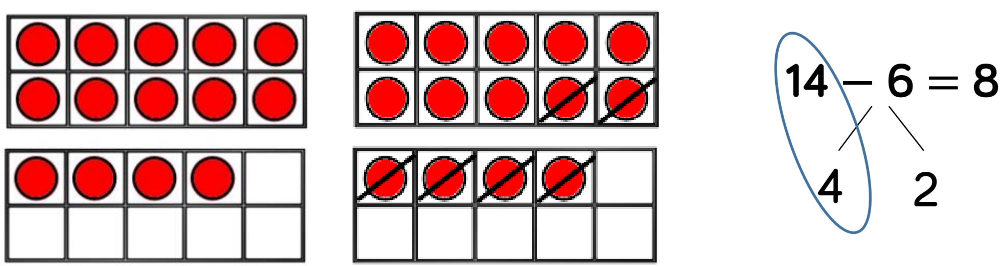
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

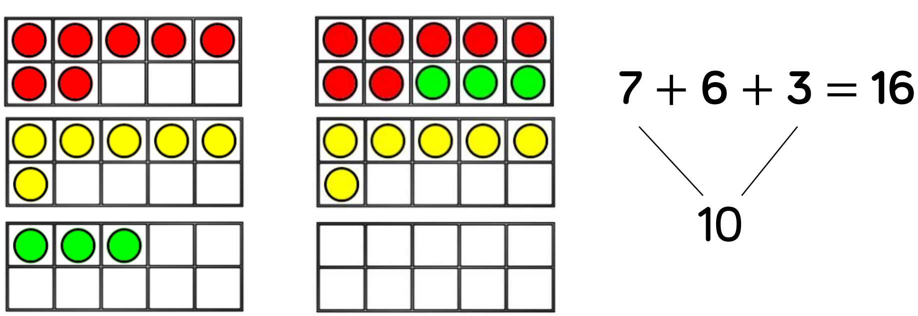
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts.
Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)







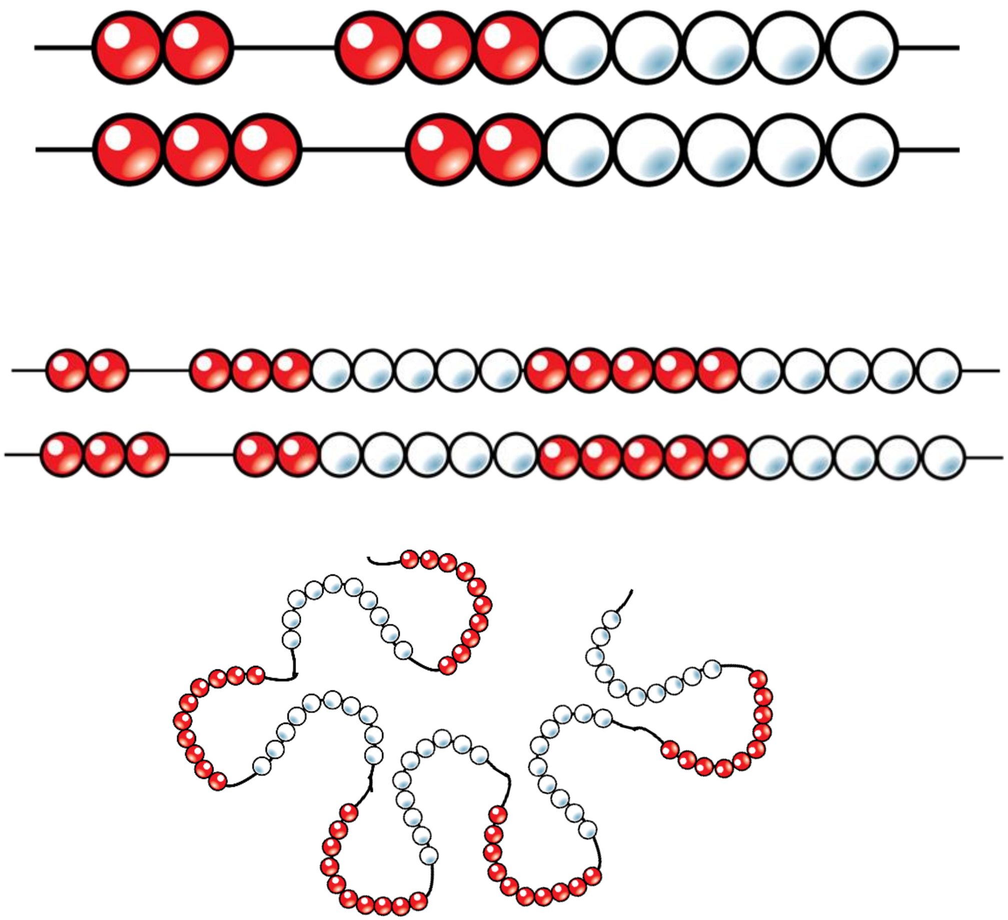
Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Bead Strings



Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

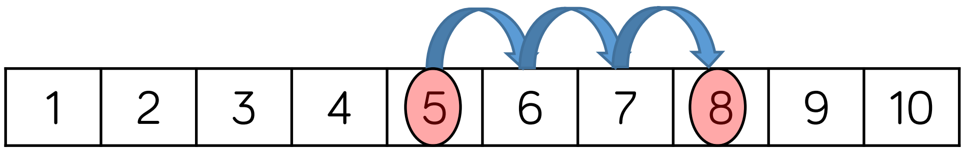
Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead, $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

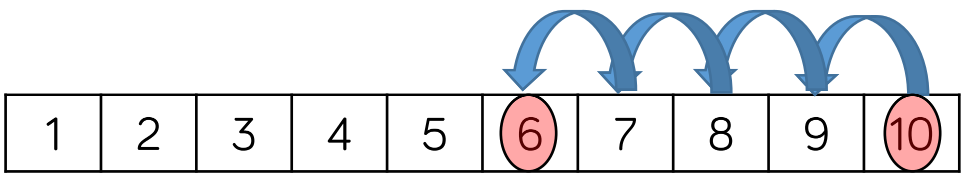
Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Number Tracks

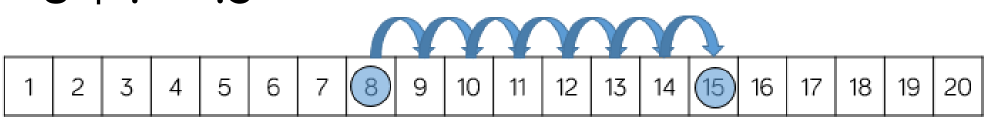
$5 + 3 = 8$



$10 - 4 = 6$



$8 + 7 = 15$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

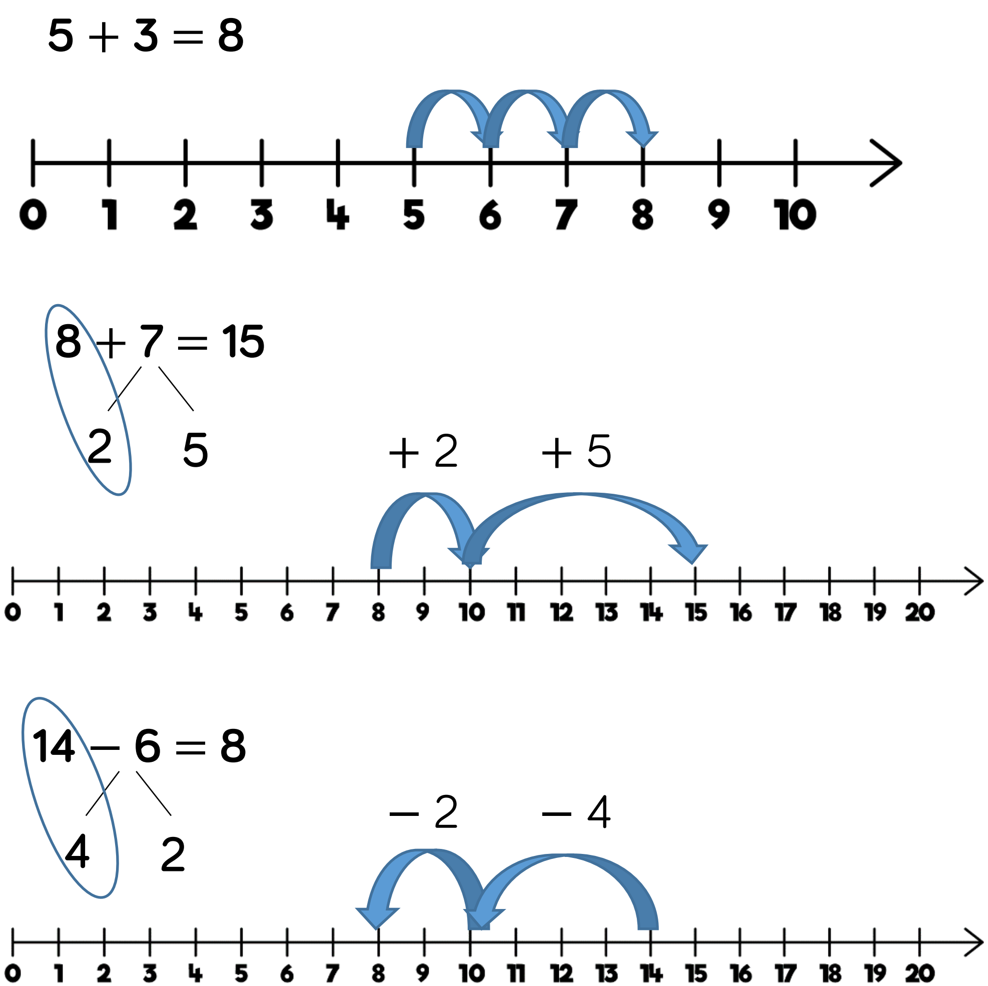
When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled)



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

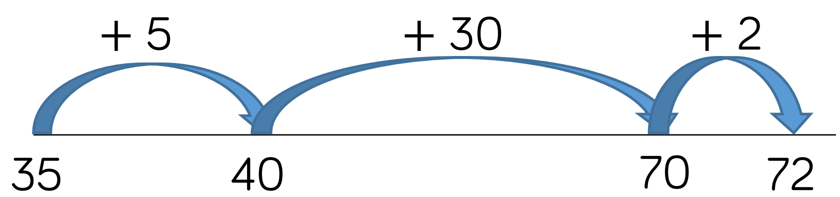
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

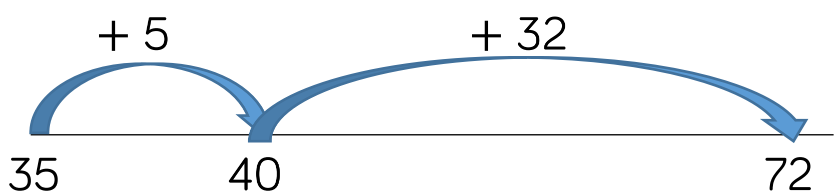
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

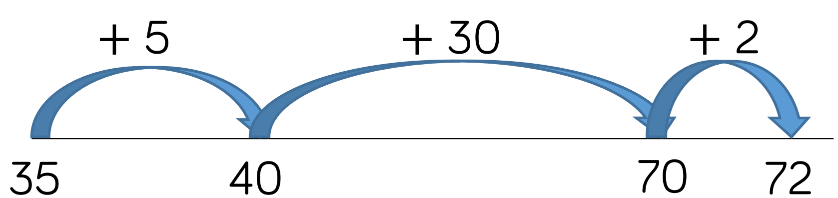
35 + 37 = 72



35 + 37 = 72



72 – 35 = 37



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

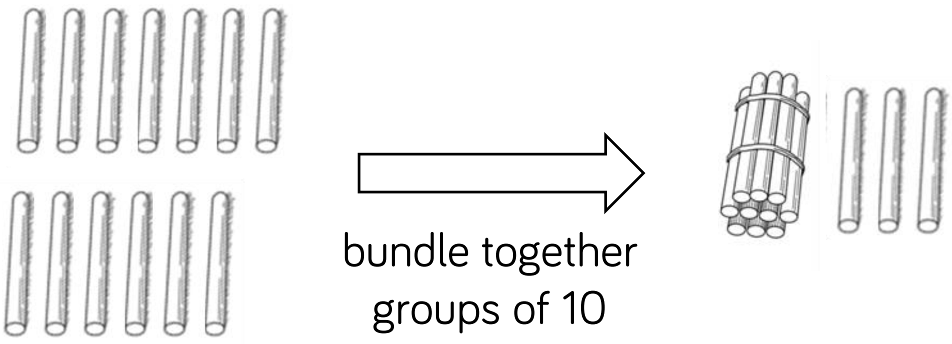
Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

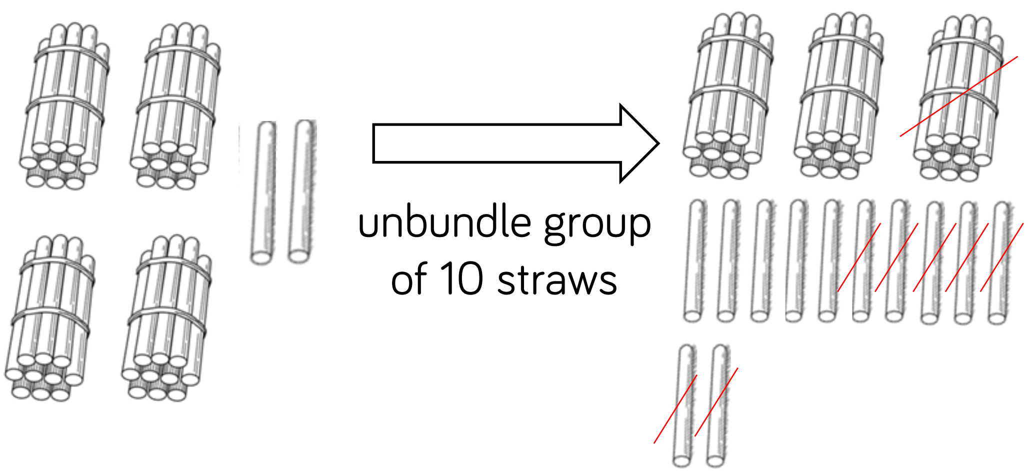
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Straws

7 + 6 = 13



42 – 17 = 25



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

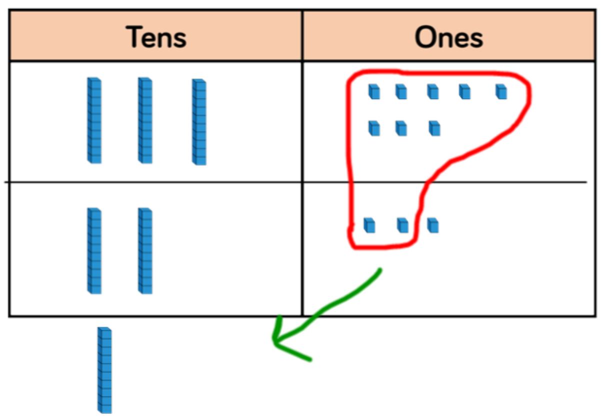
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

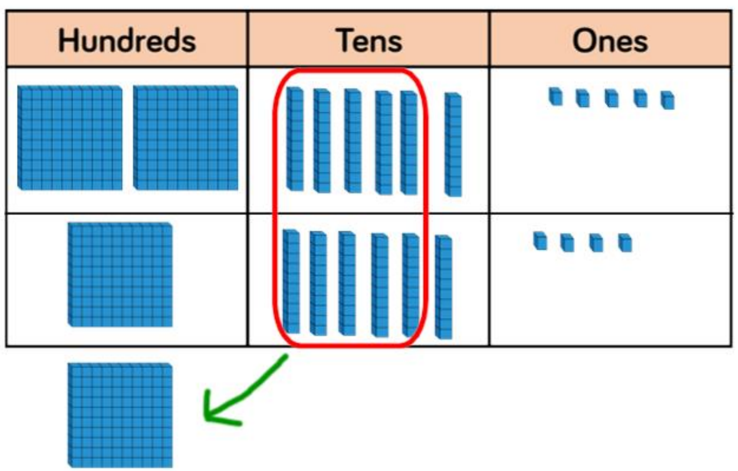
When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

Base 10/Dienes (addition)




```
   38
+  23
-----
   61
-----
  1
```




```
   265
+  164
------
   429
------
   1
```

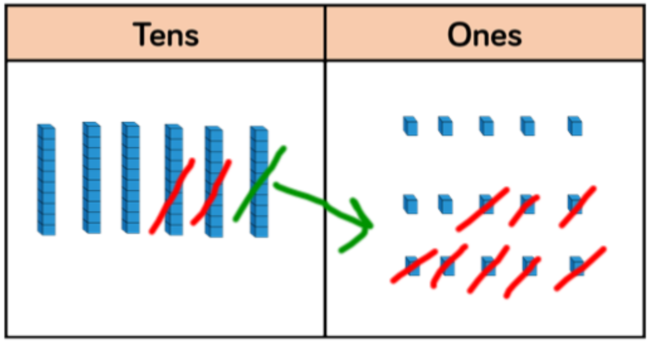
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

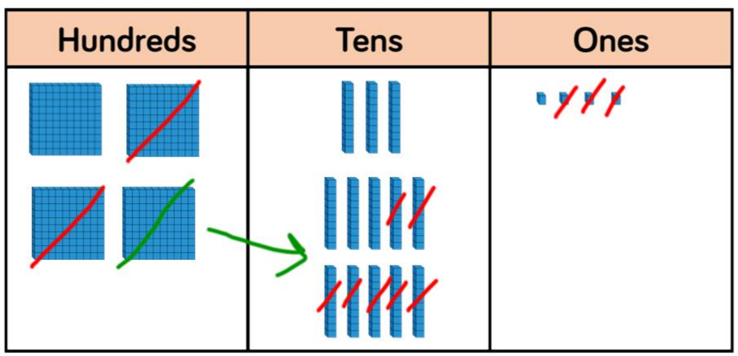
Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Base 10/Dienes (subtraction)



$$\begin{array}{r} {}^{5}\cancel{6}{}^{1}5 \\ -\ 28 \\ \hline 37 \\ \hline \end{array}$$



$$\begin{array}{r} {}^{3}\cancel{4}{}^{1}35 \\ -\ 273 \\ \hline 162 \\ \hline \end{array}$$

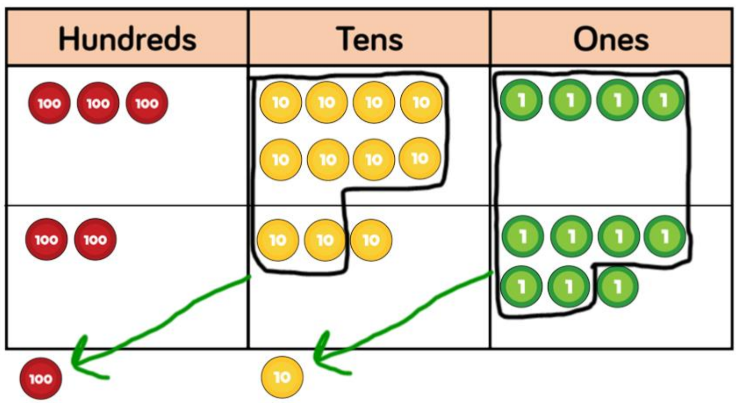
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

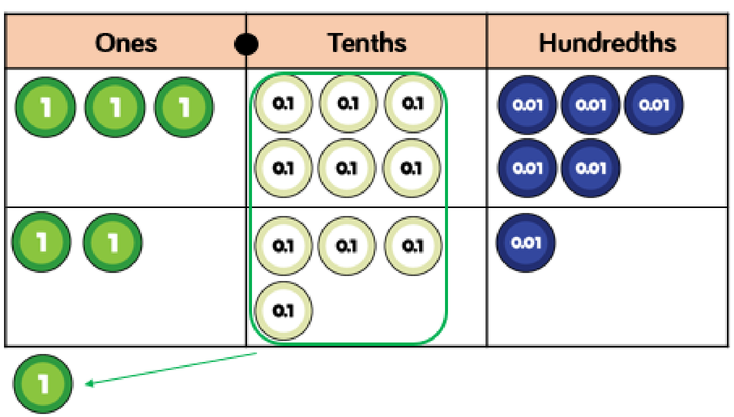
Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)




```
  384
+ 237
  621
  1 1
```




```
  3.65
+ 2.41
  6.06
  1
```

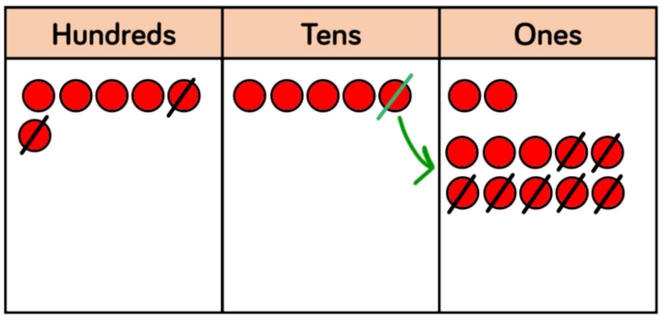
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

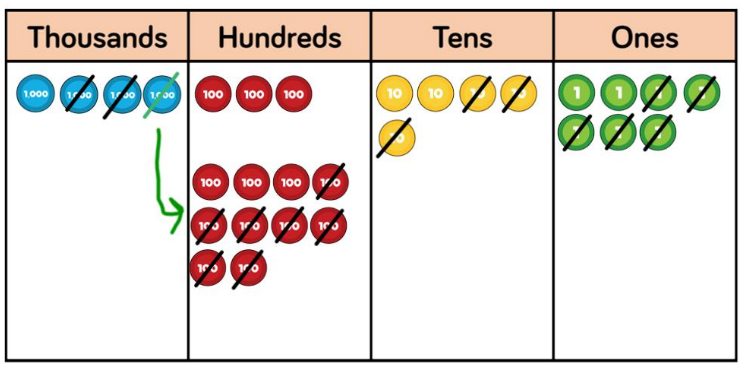
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)



$$\begin{array}{r} {}^{4}\not{5}{}^{1}\\ 6\,5\,2 \\ -\ 2\,0\,7 \\ \hline 4\,4\,5 \\ \hline \end{array}$$



$$\begin{array}{r} {}^{3}\not{4}{}^{1}3\,5\,7 \\ -\ 2\,7\,3\,5 \\ \hline 1\,6\,2\,2 \\ \hline \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

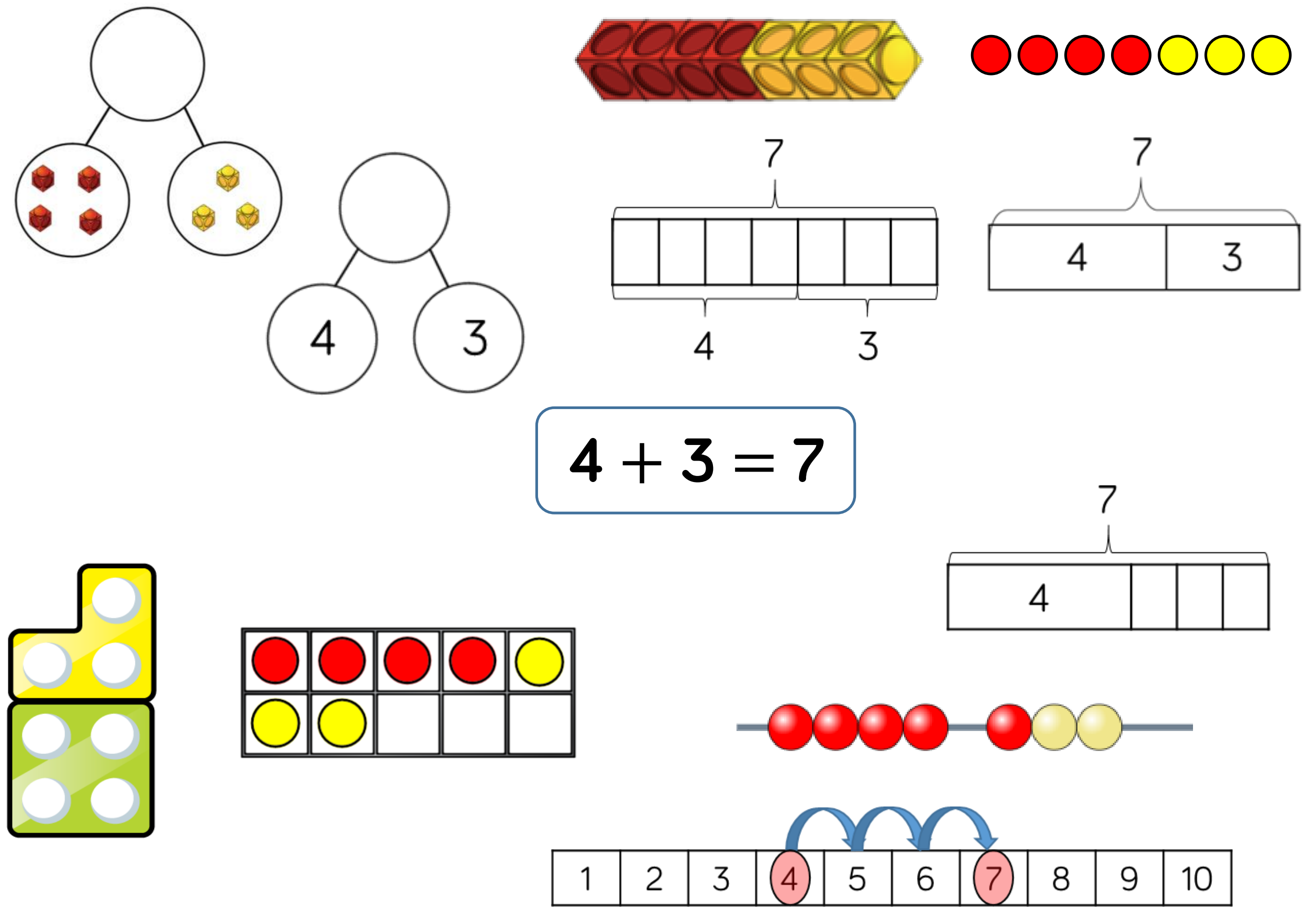
When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

Addition

Skill	Year	Representations and models	
Add two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Add 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead strings (20) Number tracks Number lines (labelled) Straws
Add three 1-digit numbers	2	Part-whole model Bar model	Ten frames (within 20) Number shapes
Add 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square

Skill	Year	Representations and models	
Add two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters
Add with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column addition
Add with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column addition
Add with more than 4 digits	5	Part-whole model Bar model	Place value counters Column addition
Add with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column addition

Skill: Add 1-digit numbers within 10	Year: 1

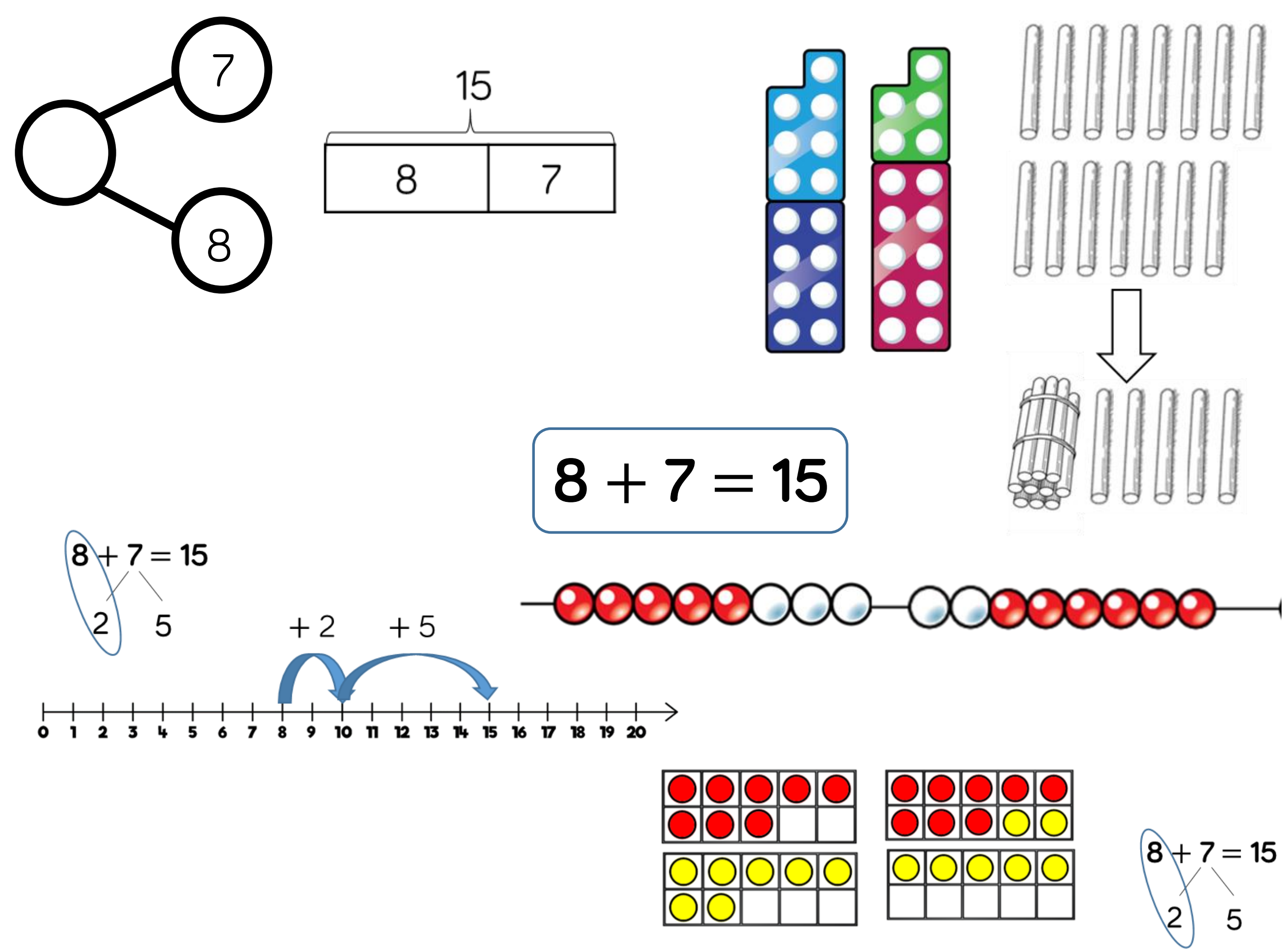


When adding numbers to 10, children can explore both aggregation and augmentation.

The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation.

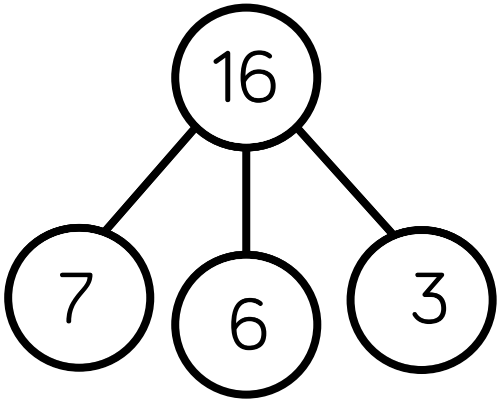
The combination bar model, ten frame, bead string and number track all support augmentation.

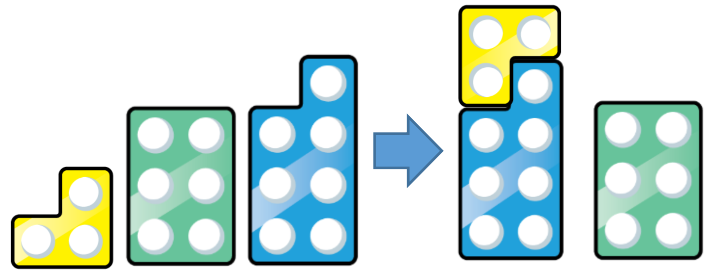
Skill: Add 1 and 2-digit numbers to 20	Year: 1/2



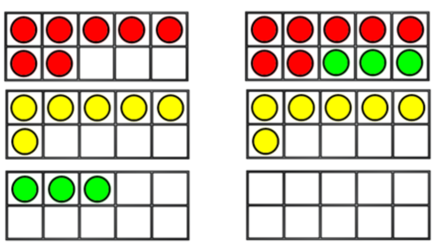
When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. In Year 1, this is only done just by counting on. From Year 2, use different manipulatives can be used to represent this exchange alongside number lines to support children in understanding how to partition their jumps.

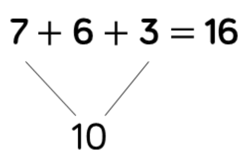
Skill: Add three 1-digit numbers	Year: 2

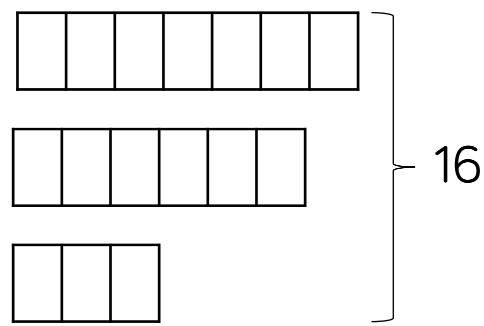




$$7 + 6 + 3 = 16$$





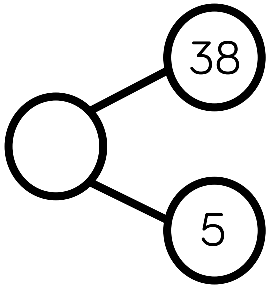


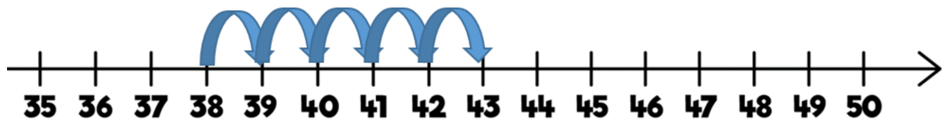
When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently.

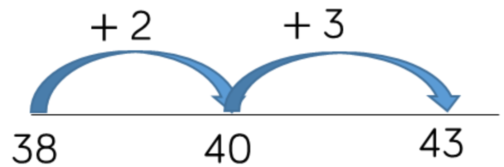
This supports children in their understanding of commutativity.

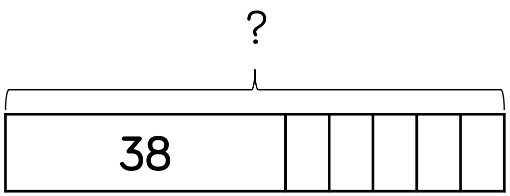
Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

Skill: Add 1-digit and 2-digit numbers to 100	Year: 2/3

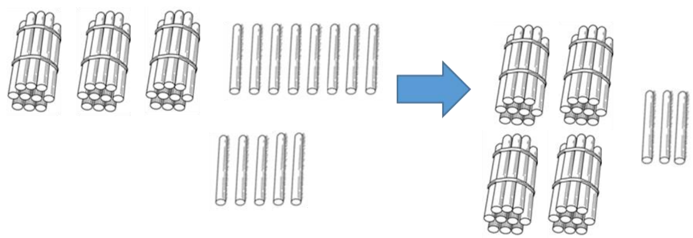








$38 + 5 = 43$

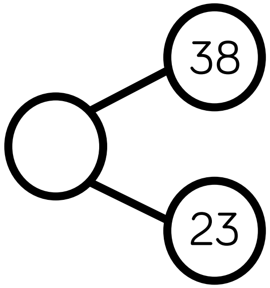
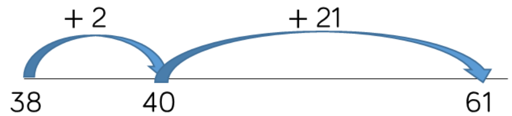
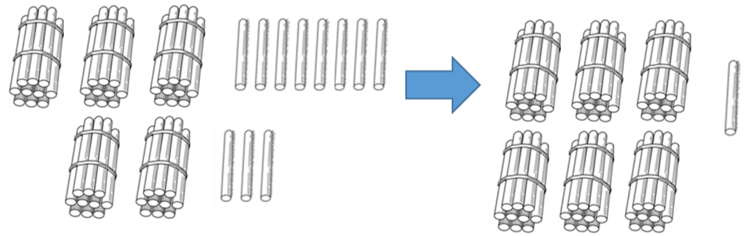
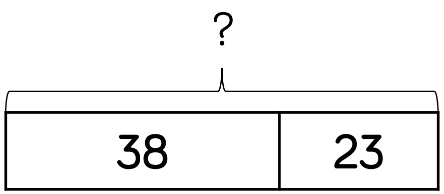
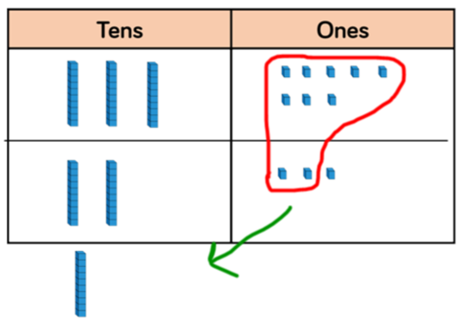
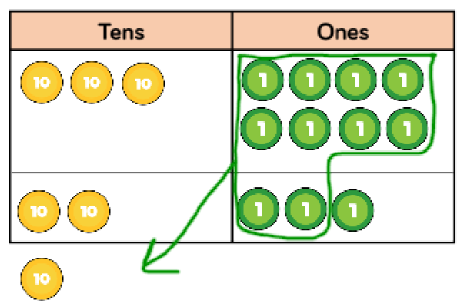


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

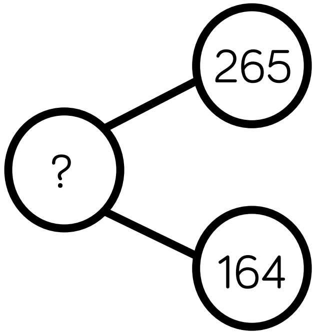
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

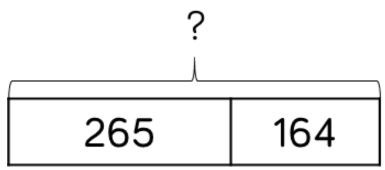
They should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$.

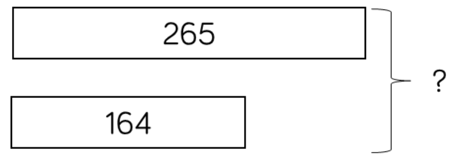
Hundred squares and straws can support children to find the number bond to 10.

Skill: Add two 2-digit numbers to 100	Year: 2/3
    **38 + 23 = 61**  $\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$ 	Children can use a blank number line and other representations to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient. From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

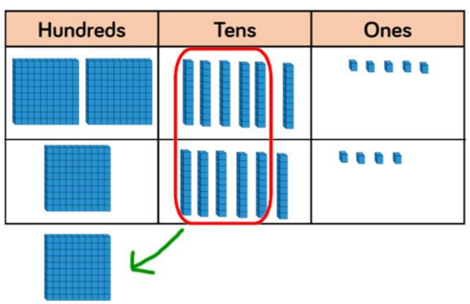
Skill: Add numbers with up to 3 digits	Year: 3



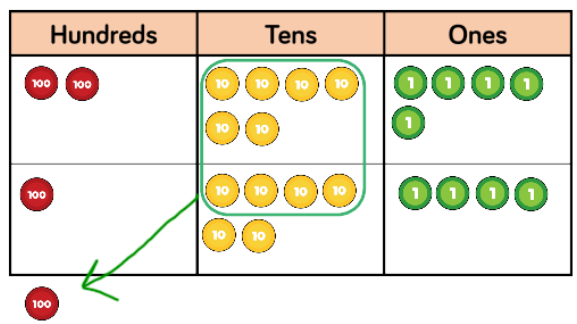




265 + 164 = 429




```
  265
+ 164
  429
   1
```

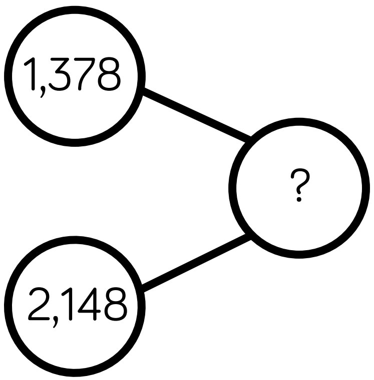


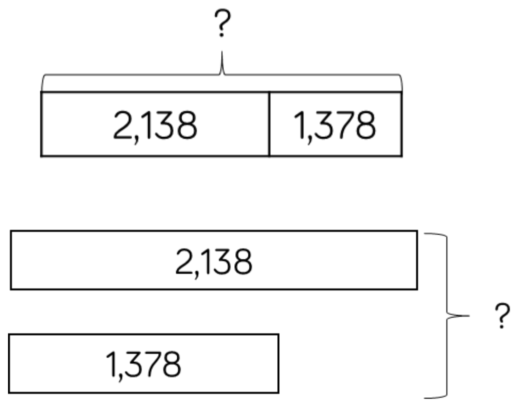
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

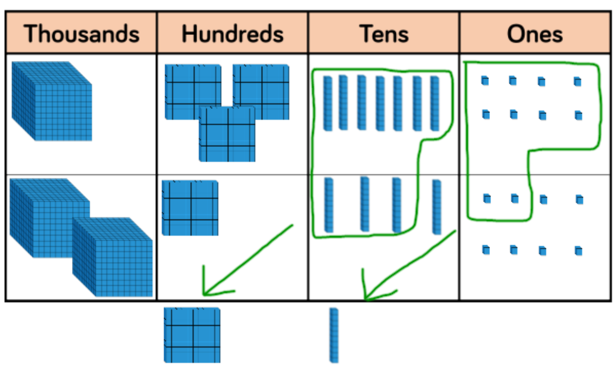
Skill: Add numbers with up to 4 digits	Year: 4

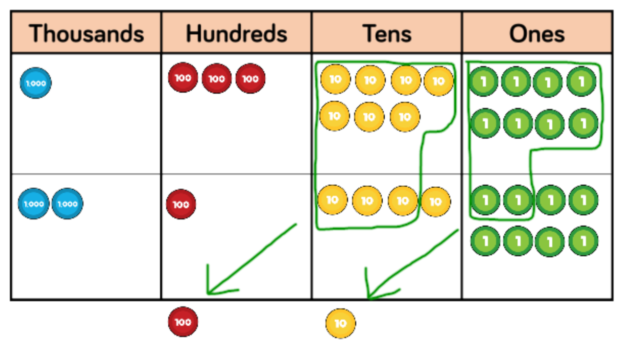




	1	3	7	8
+	2	1	4	8
	3	5	2	6
		1	1	

1,378 + 2,148 = 3,526



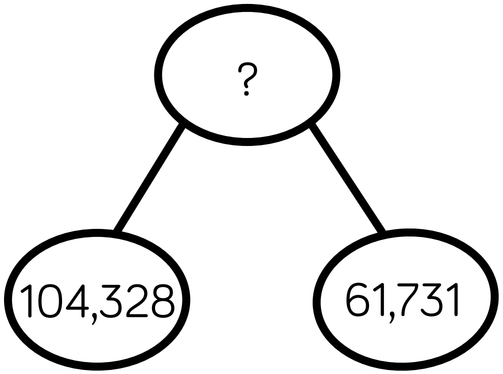


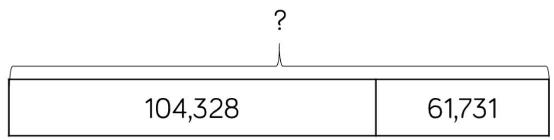
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

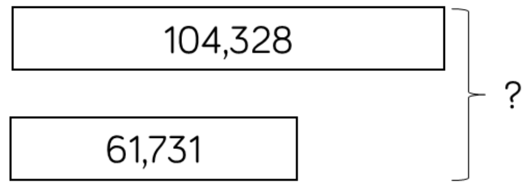
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

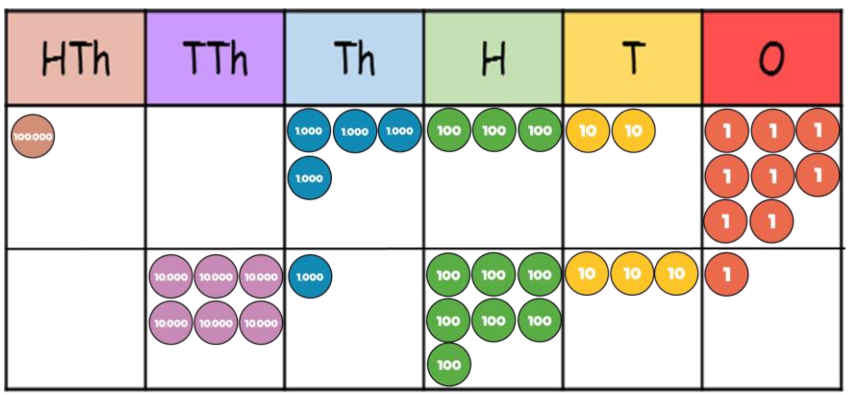
Skill: Add numbers with more than 4 digits	Year: 5/6







104,328 + 61,731 = 166,059

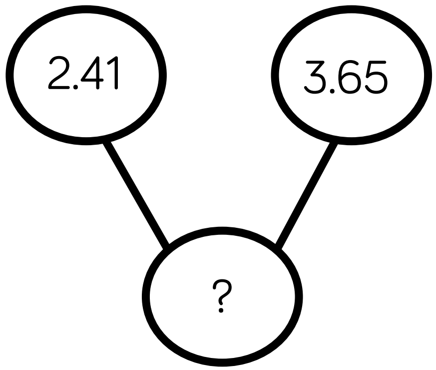


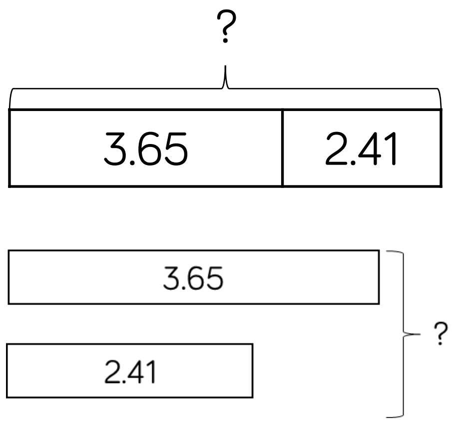
1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
		1			

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

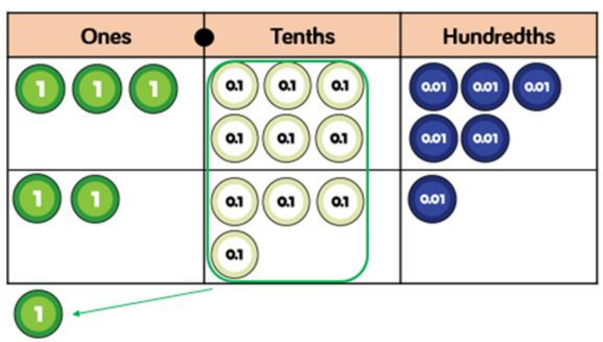
Skill: Add with up to 3 decimal places	Year: 5

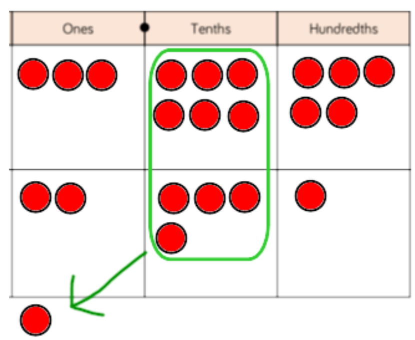




$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline {\scriptstyle 1} \end{array}$$

3.65 + 2.41 = 6.06





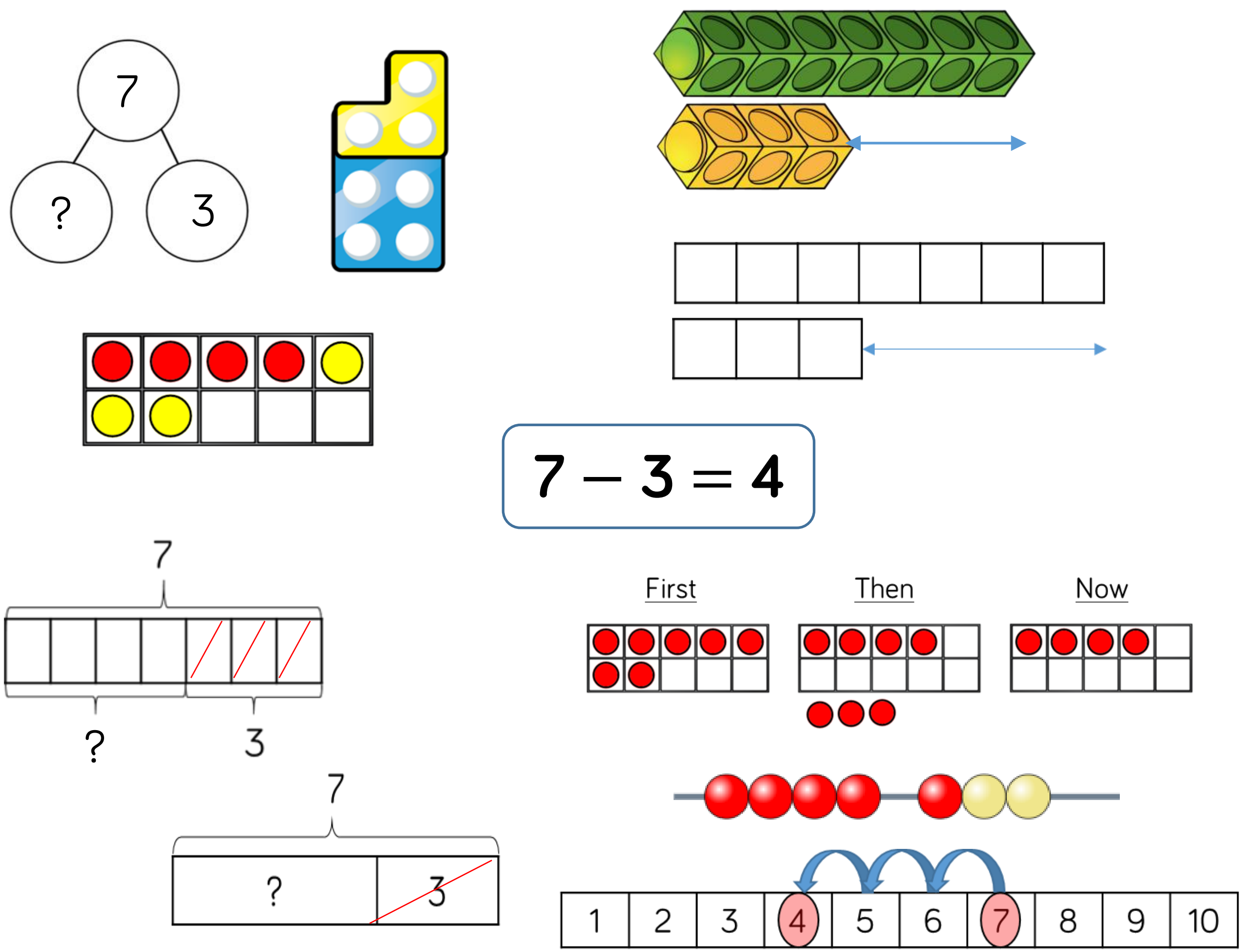
Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.

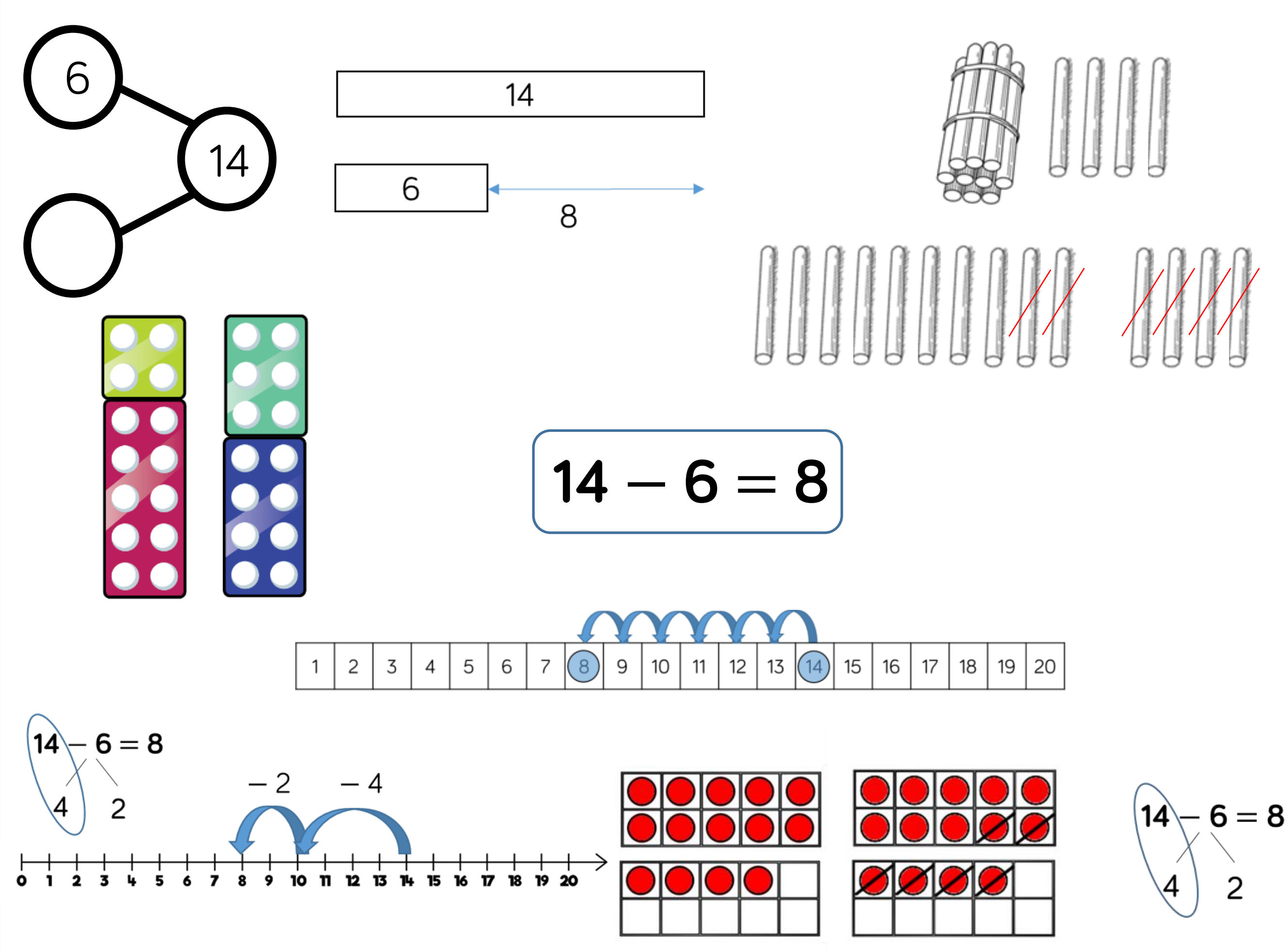
Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

Subtraction

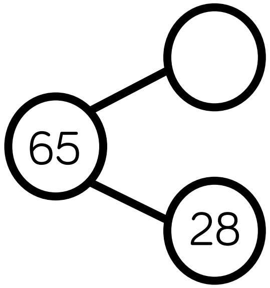
Skill	Year	Representations and models	
Subtract two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Subtract 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead string (20) Number tracks Number lines (labelled) Straws
Subtract 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square
Subtract two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters

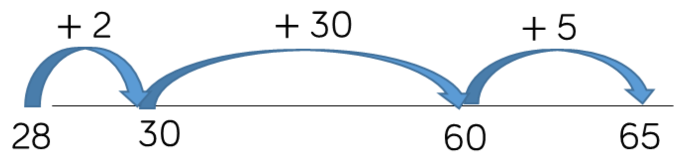
Skill	Year	Representations and models	
Subtract with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column subtraction
Subtract with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column subtraction
Subtract with more than 4 digits	5	Part-whole model Bar model	Place value counters Column subtraction
Subtract with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column subtraction

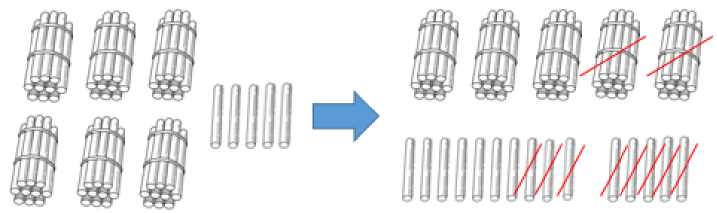
Skill: Subtract 1-digit numbers within 10	Year: 1
	Part-whole models, bar models, ten frames and number shapes support partitioning. Ten frames, number tracks, single bar models and bead strings support reduction. Cubes and bar models with two bars can support finding the difference.

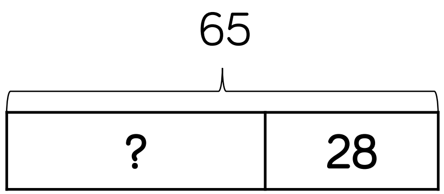
Skill: Subtract 1 and 2-digit numbers to 20	Year: 1/2
	In Year 1, subtracting one-digit numbers that cross 10, is done by counting back, using objects, number tracks and number lines. From Year 2, children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

Skill: Subtract 1 and 2-digit numbers to 100	Year: 2/3

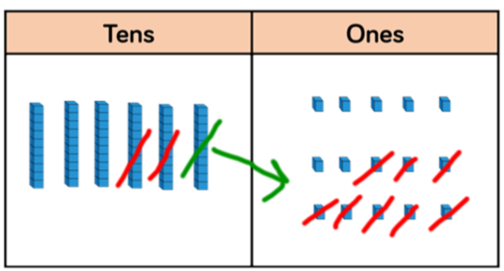




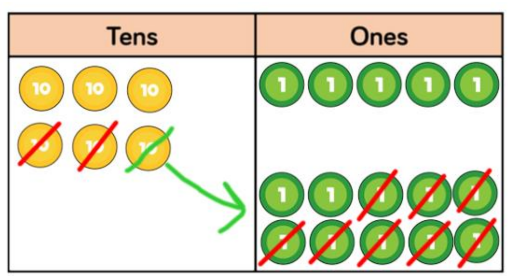




65 – 28 = 37

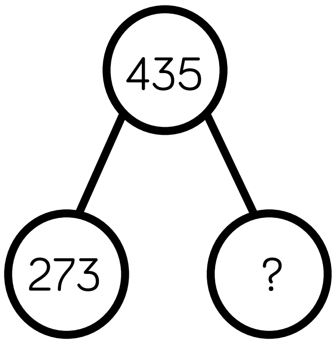


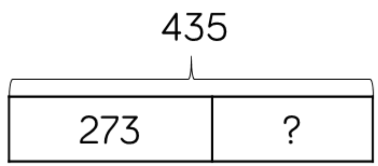
$$\begin{array}{r} {}^{5}\cancel{6}{}^{1}5 \\ -\ 28 \\ \hline 37 \\ \hline \end{array}$$

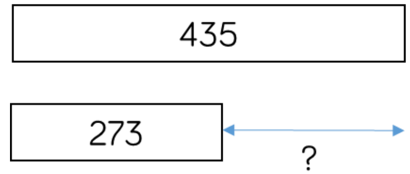


Children can also use a blank number line to count back to find the difference.
Encourage them to jump to multiples of 10 to become more efficient.
From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters.
As numbers become larger, straws become less efficient.

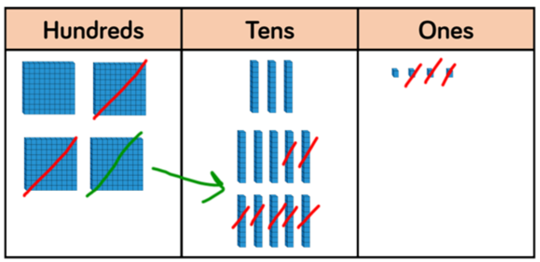
Skill: Subtract numbers with up to 3 digits	Year: 3



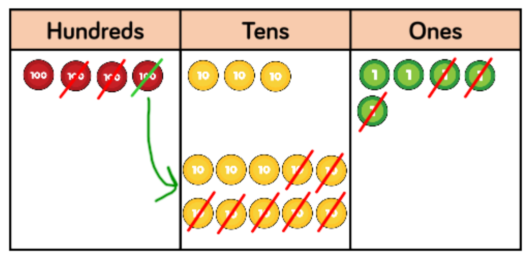




435 − 273 = 162



$$\begin{array}{r} 435 \\ -\ 273 \\ \hline 162 \\ \hline \end{array}$$

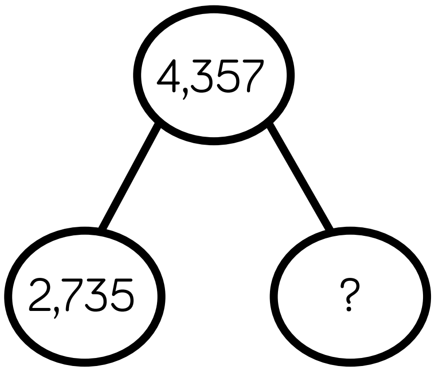


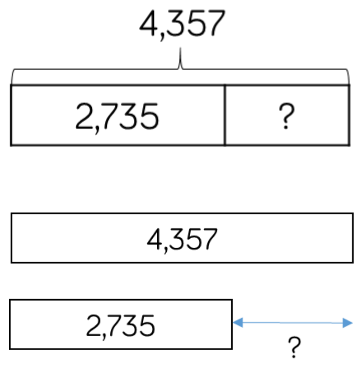
Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

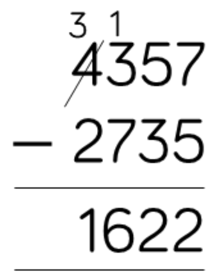
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

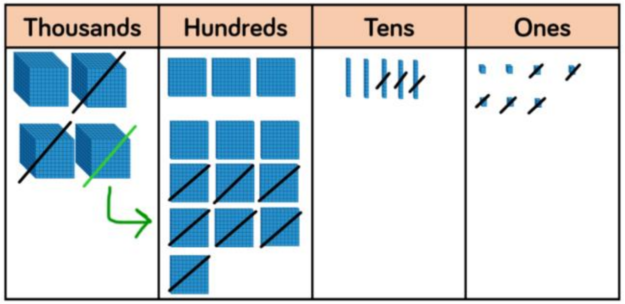
Skill: Subtract numbers with up to 4 digits	Year: 4

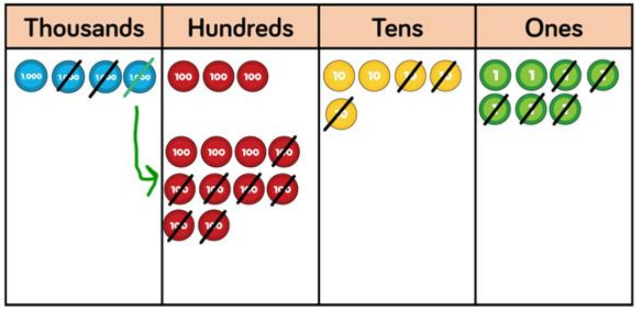






4,357 – 2,735 = 1,622



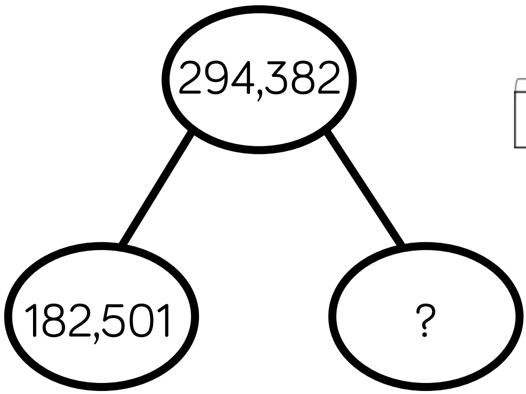


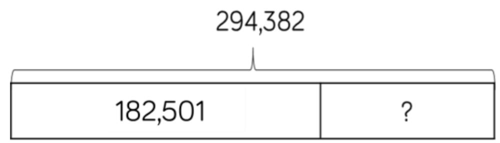
Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

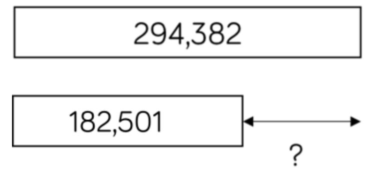
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

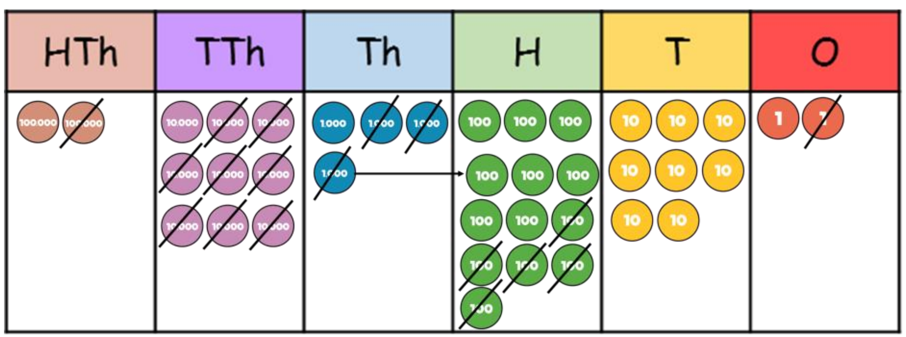
Skill: Subtract numbers with more than 4 digits	Year: 5/6







294,382 – 182,501 = 111,881

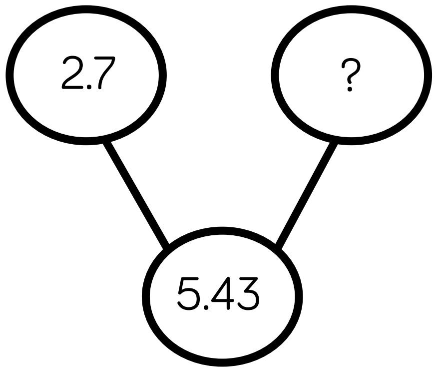


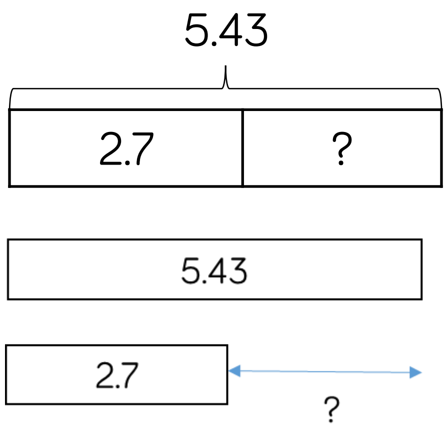
	2	9	$\overset{3}{\cancel{4}}$	$^{1}3$	8	2
–	1	8	2	5	0	1
	1	1	1	8	8	1

Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

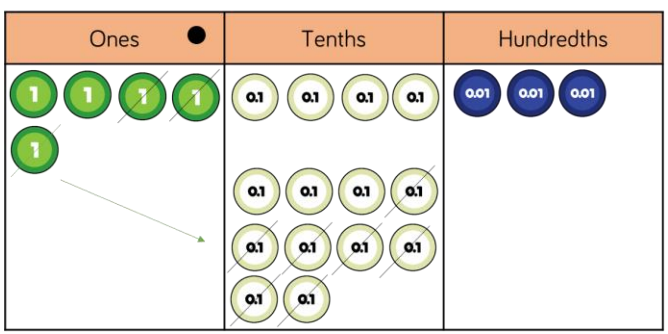
Skill: Subtract with up to 3 decimal places	Year: 5/6

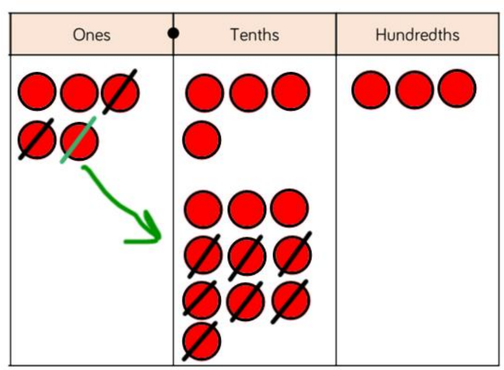




$$\begin{array}{r} \overset{4}{\not{5}}.\overset{1}{4}3 \\ -\ 2.7 \\ \hline 2.73 \end{array}$$

5.43 – 2.7 = 2.73





Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total – The aggregate or the sum found by addition.

Year 1 - 6

Calculation Policy

Multiplication and Division

#MathsEveryoneCan

White Rose Maths

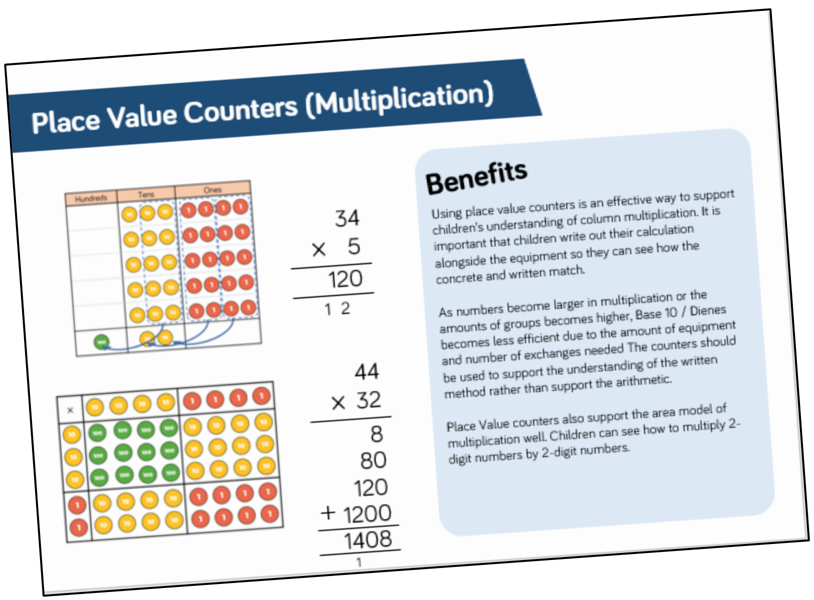
Notes and Guidance

Calculation Policy

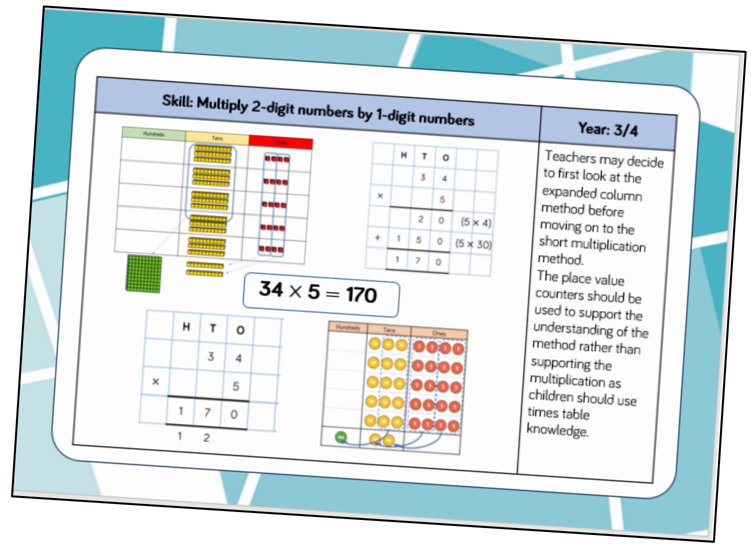
Welcome to the White Rose Maths Calculation Policy.

This document is broken down into addition and subtraction, and multiplication and division.

At the start of each policy, there is an overview of the different models and images that can support the teaching of different concepts. These provide explanations of the benefits of using the models and show the links between different operations.

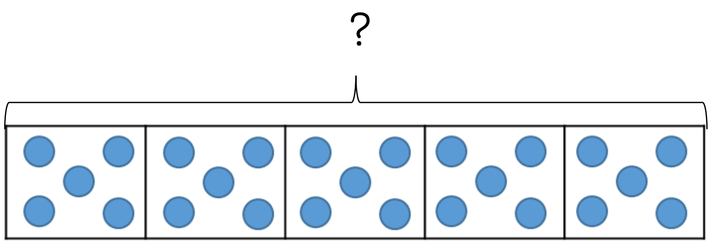


Each operation is then broken down into skills and each skill has a dedicated page showing the different models and images that could be used to effectively teach that concept.

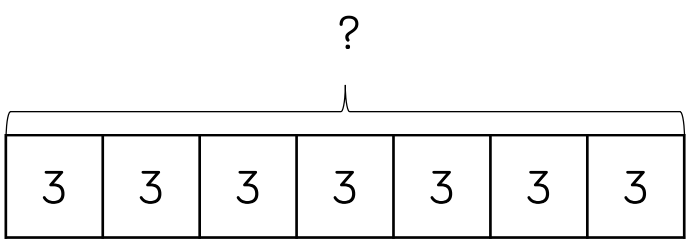


There is an overview of skills linked to year groups to support consistency through out school. A glossary of terms is provided at the end of the calculation policy to support understanding of the key language used to teach the four operations.

Bar Model

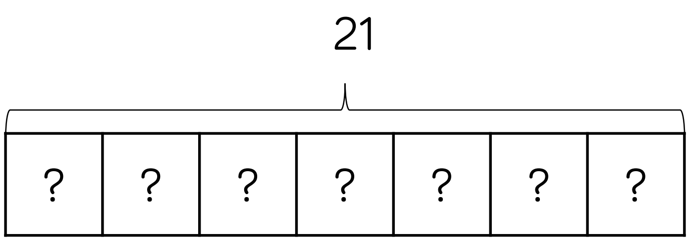


$5 \times 5 = 25$

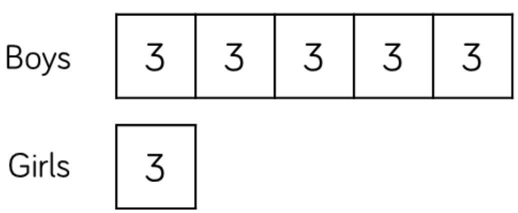


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits

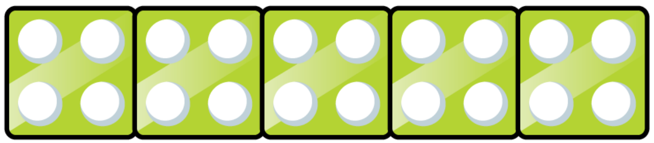
Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

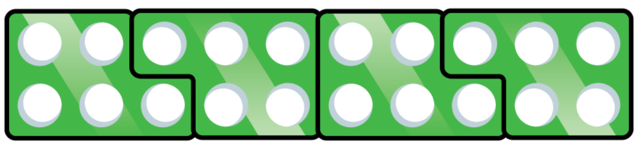
It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

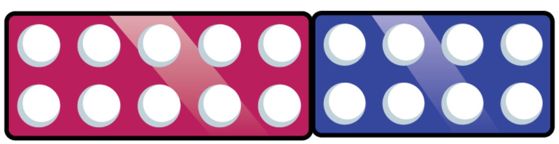
Number Shapes



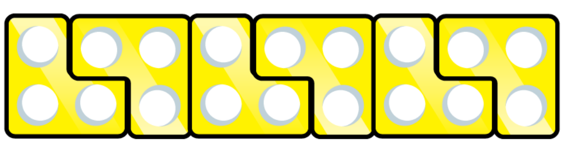
$5 \times 4 = 20$
$4 \times 5 = 20$



$5 \times 4 = 20$
$4 \times 5 = 20$



$18 \div 3 = 6$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

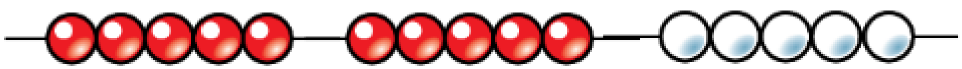
Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd × odd = even, odd × even = odd, even × even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Bead Strings

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$



$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

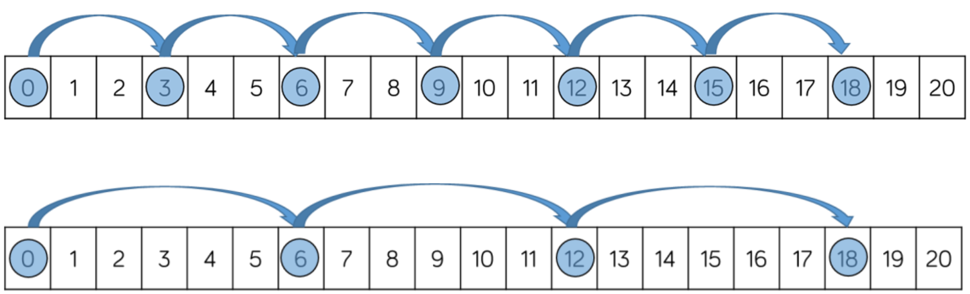
Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

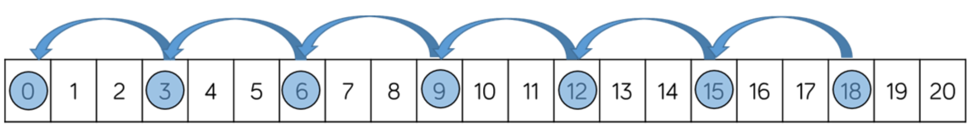
Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Tracks



$$6 \times 3 = 18$$
$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

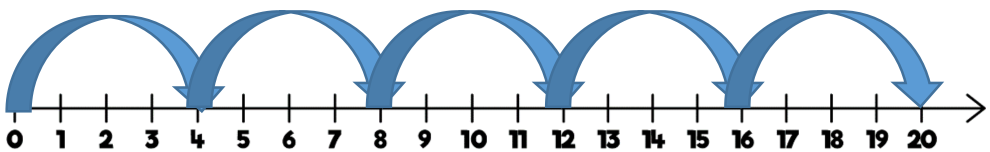
Benefits

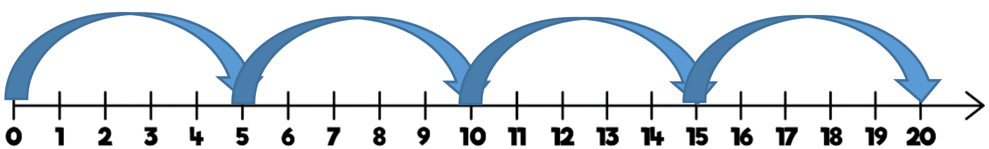
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

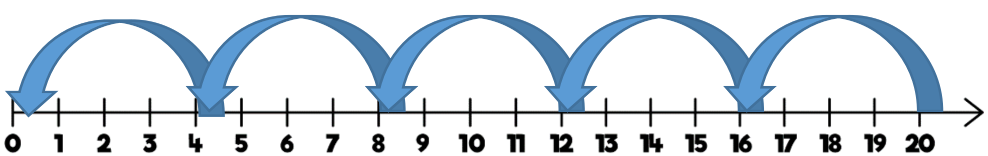
Number Lines (labelled)





$4 \times 5 = 20$

$5 \times 4 = 20$



$20 \div 4 = 5$

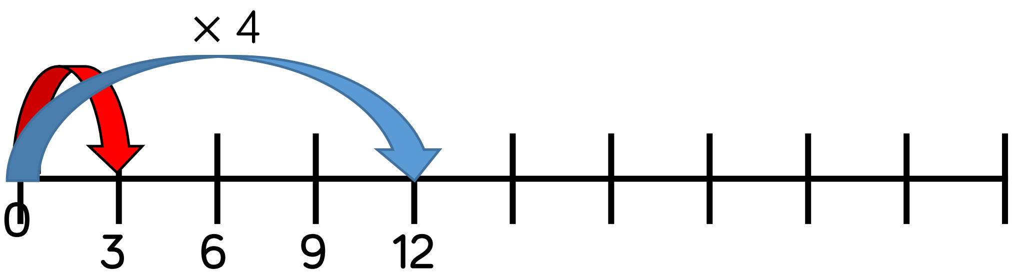
Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

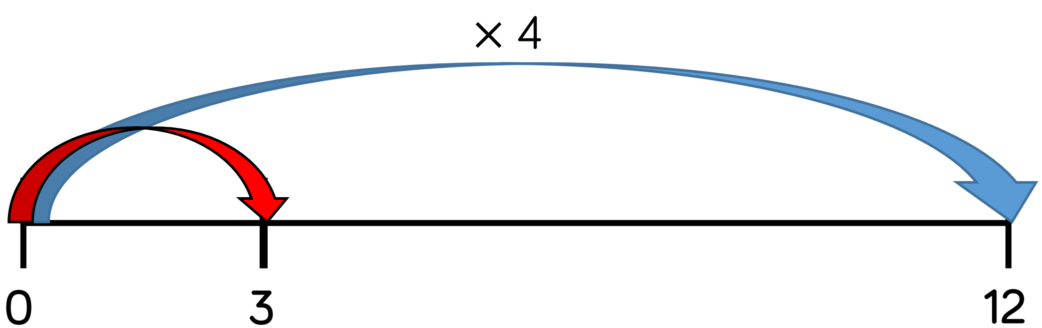
When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

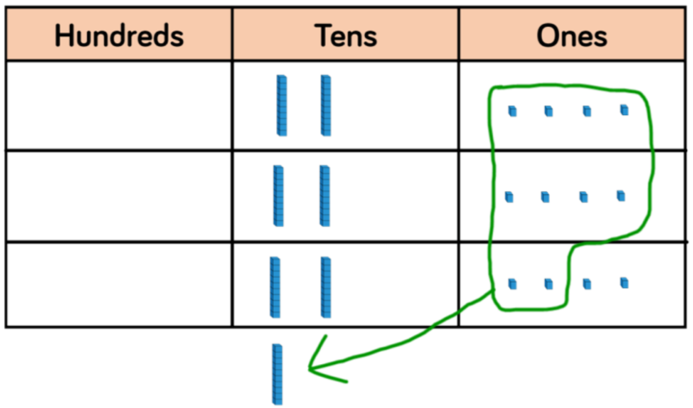
Benefits

Children can use blank number lines to represent scaling as multiplication or division.

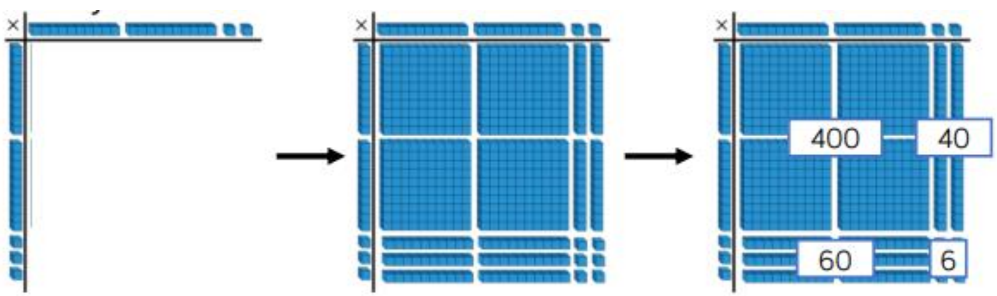
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)



$$\begin{array}{r} 24 \\ \times \quad 3 \\ \hline 72 \\ \hline {}_{1} \end{array}$$



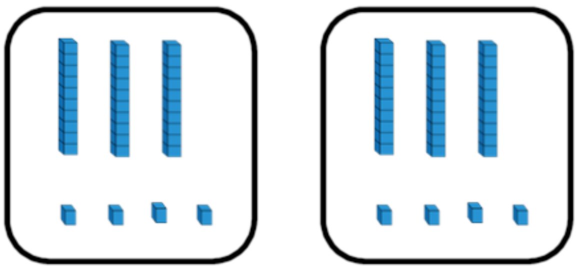
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

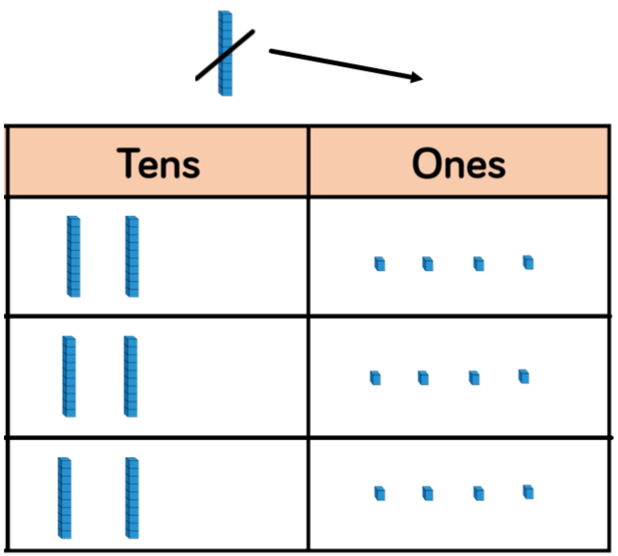
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

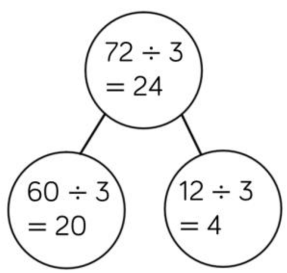
Base 10/Dienes (division)



$$68 \div 2 = 34$$



$$72 \div 3 = 24$$



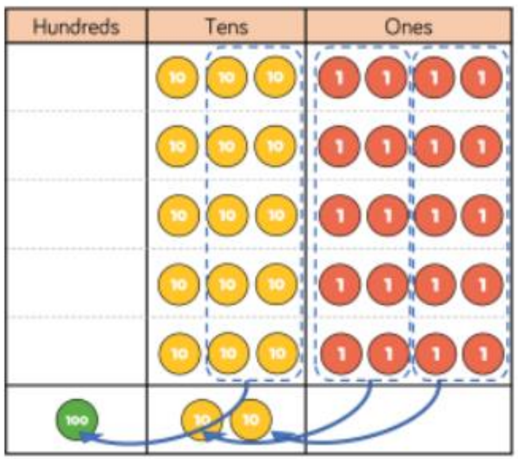
Benefits

Using Base 10 or Dienes is an effective way to support children’s understanding of division.

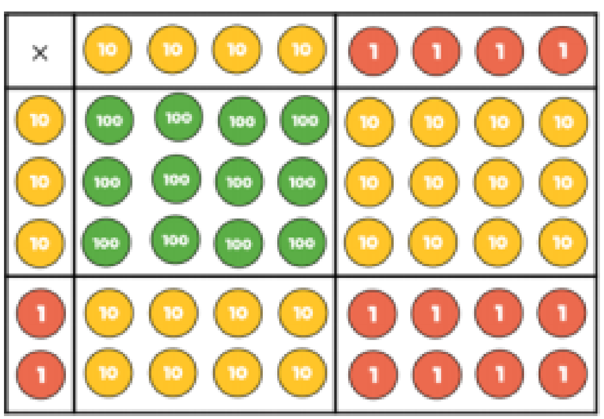
When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (multiplication)




```
   34
×   5
-----
  170
-----
 1 2
```




```
    44
×   32
------
     8
    80
   120
+ 1200
------
  1408
------
   1
```

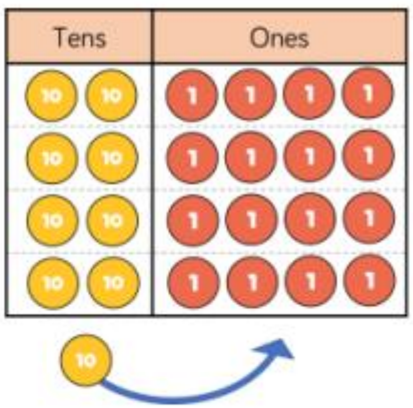
Benefits

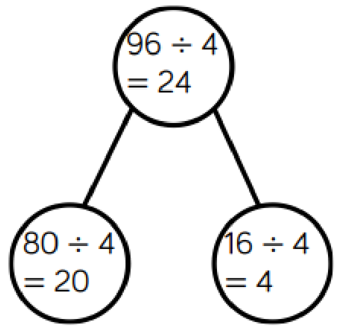
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

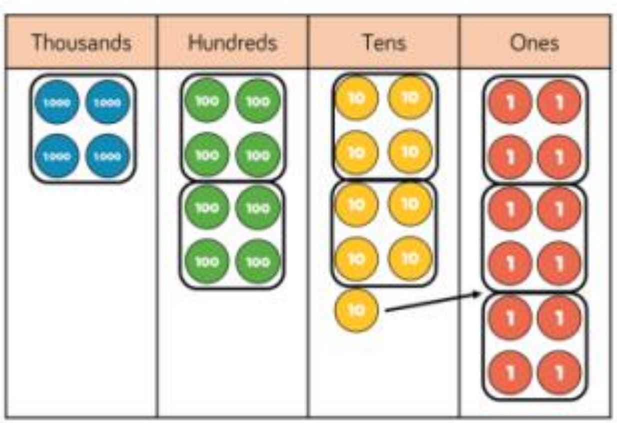
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division)







$$\begin{array}{r|l} & 1223 \\ \hline 4 & 489^{1}2 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

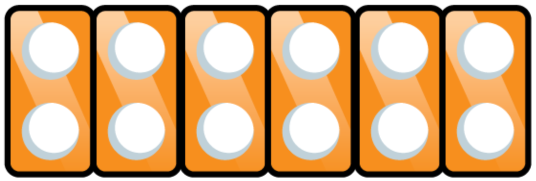
Times Tables

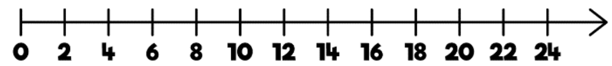
Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 2-times table	2	Bar model Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 5-times table	2	Bar model Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 10-times table	2	Hundred square Number shapes Counters Money	Ten frames Bead strings Number lines Base 10

Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 3-times table	3	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 4-times table	3	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 8-times table	3	Hundred square Number shapes	Bead strings Number tracks Everyday objects
Recall and use multiplication and division facts for the 6-times table	4	Hundred square Number shapes	Bead strings Number tracks Everyday objects

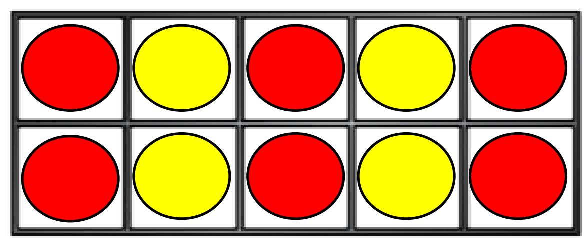
Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 7-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 9-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 11-times table	4	Hundred square Base 10	Place value counters Number lines
Recall and use multiplication and division facts for the 12-times table	4	Hundred square Base 10	Place value counters Number lines

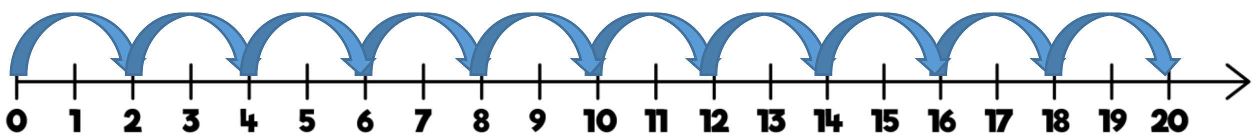
Skill: 2 times table	Year: 2





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



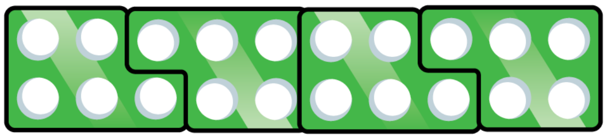


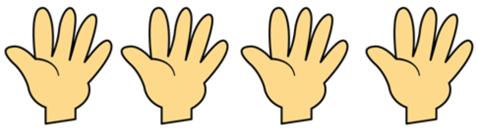
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.

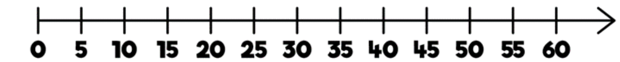
Use different models to develop fluency.

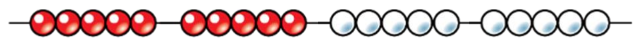
Skill: 5 times table	Year: 2

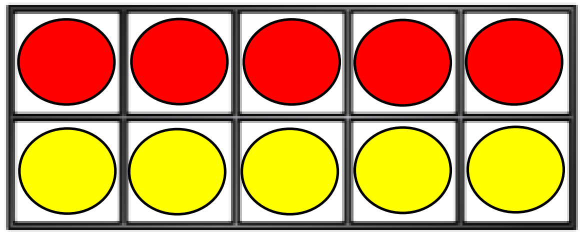


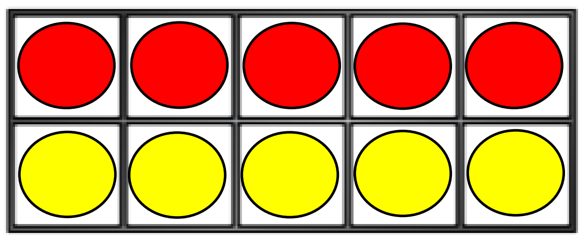


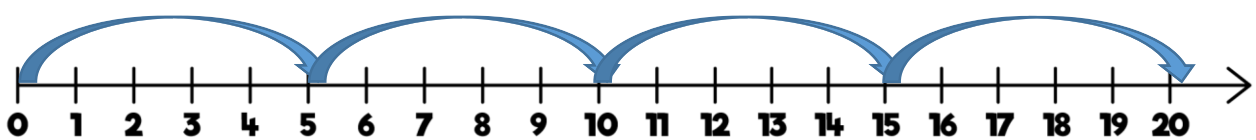
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50







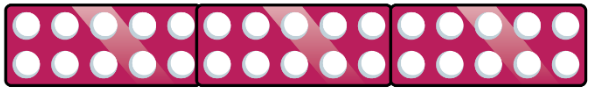


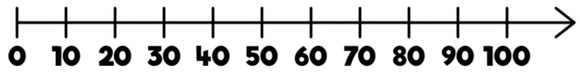


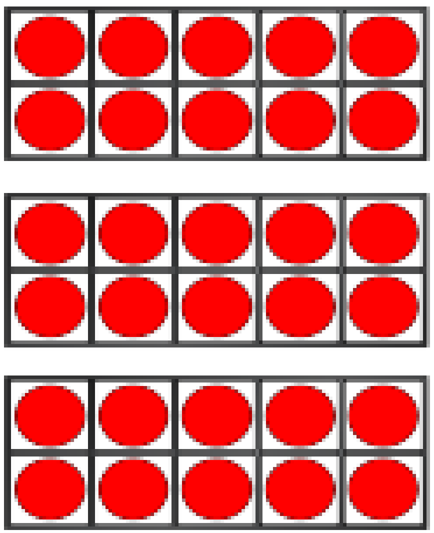
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

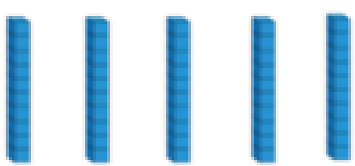
Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.

Skill: 10 times table	Year: 2







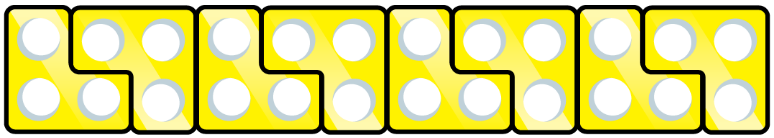


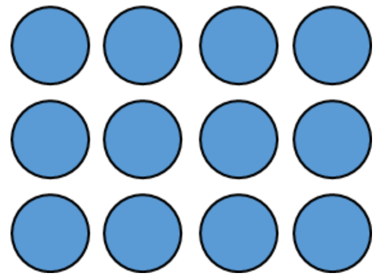
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

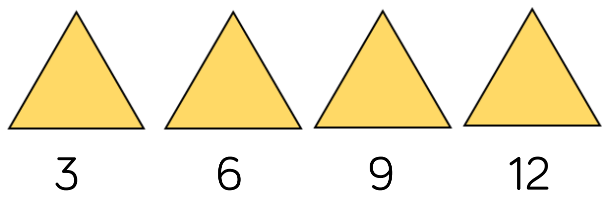
Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits- the ones are always 0, and the tens increase by 1 ten each time.

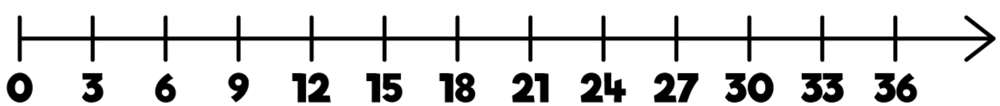
Skill: 3 times table	Year: 3





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

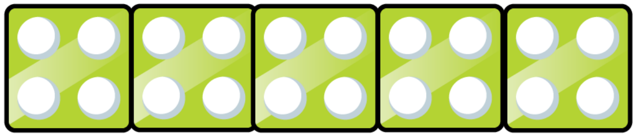




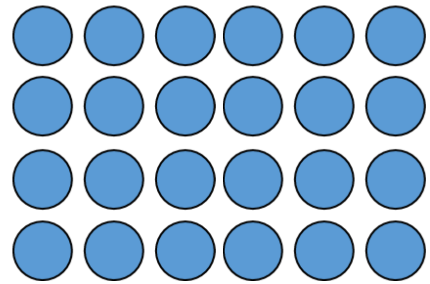
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

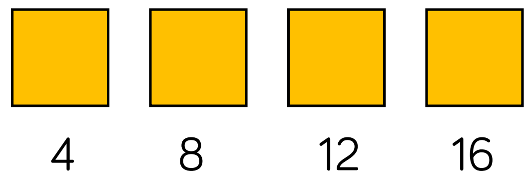
Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.

Skill: 4 times table	Year: 3

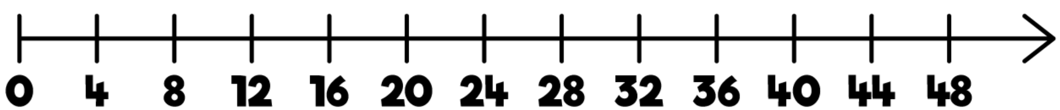


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



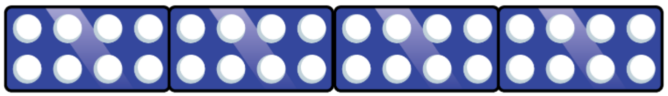


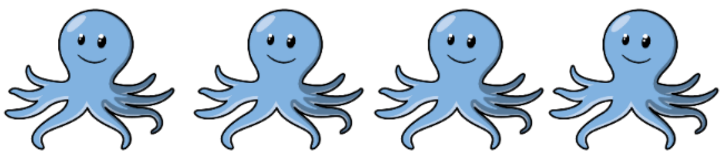
4	8	12	16	20
24	28	32	36	40
44	48	52	56	60



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the four times table, using manipulatives to support. Make links to the 2 times table, seeing how each multiple is double the twos. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Skill: 8 times table	Year: 3

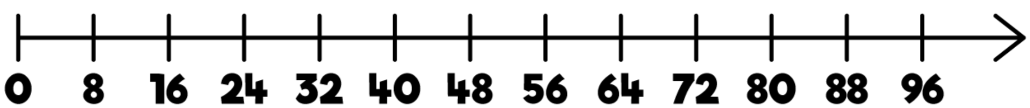




8 16 24 32

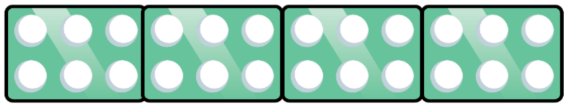
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

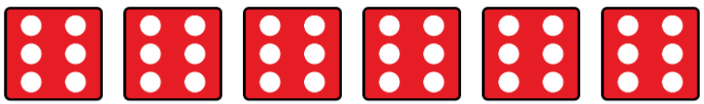
8	16	24	32	40
48	56	64	72	80



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

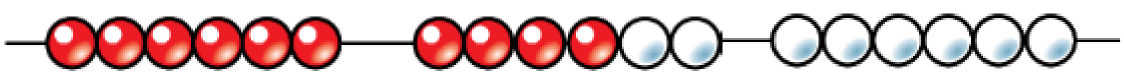
Skill: 6 times table	Year: 4

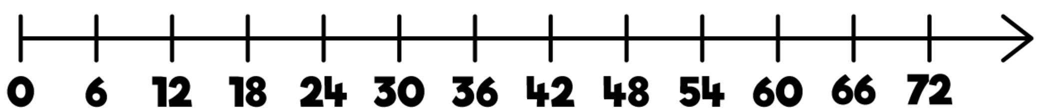




6	12	18	24	30
36	42	48	54	60
66	72	78	84	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



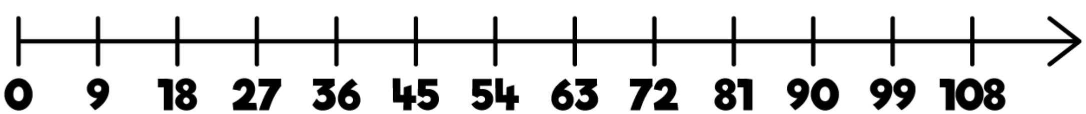


Encourage daily counting in multiples, supported by a number line or a hundred square.
Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples.
Highlight that all the multiples are even using number shapes to support.

Skill: 9 times table	Year: 4

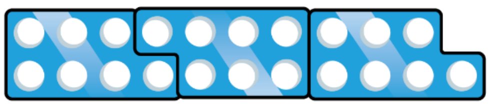
9	18	27	36	45
54	63	72	81	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



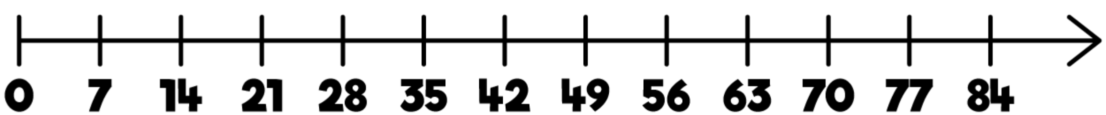
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.

Skill: 7 times table	Year: 4



7	14	21	28	35
42	49	56	63	70

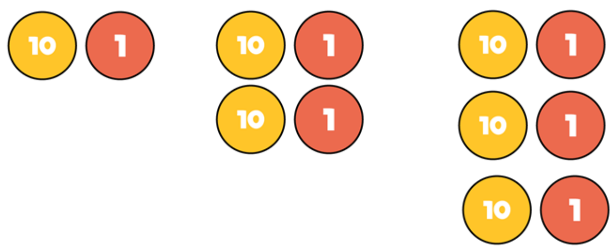
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



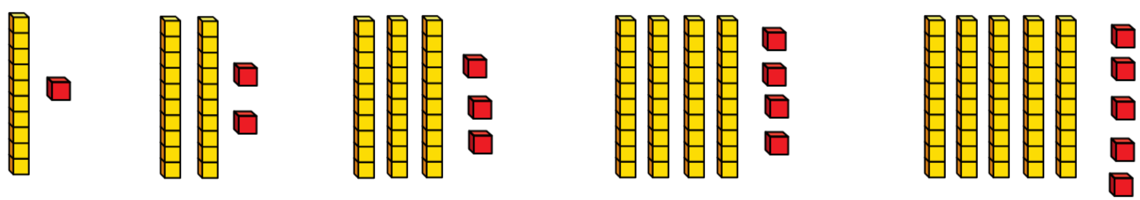
Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square. The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity. Children can still see the odd, even pattern in the multiples using number shapes to support.

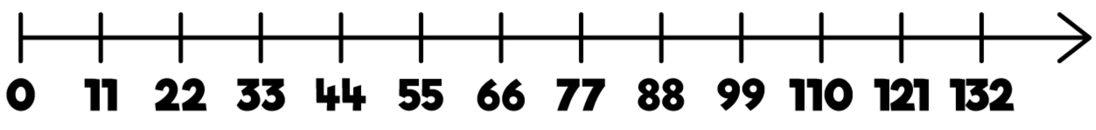
Skill: 11 times table	Year: 4

11	22	33	44	55	66
77	88	99	110	121	132



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



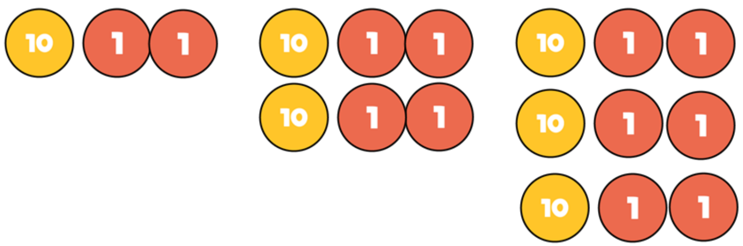


Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

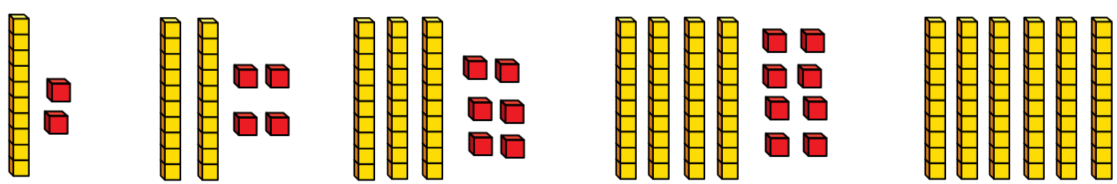
Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100

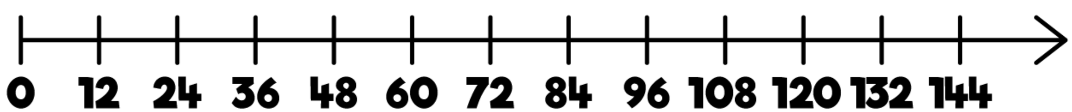
Skill: 12 times table	Year: 4

12	24	36	48	60
72	84	96	108	120
132	144			



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100





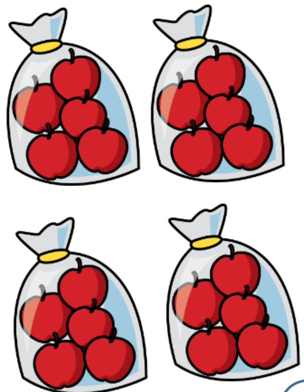
Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.

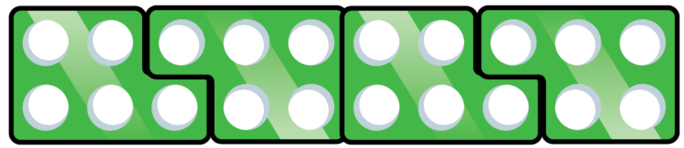
Multiplication

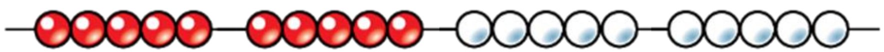
Skill	Year	Representations and models	
Solve one-step problems with multiplication	1/2	Bar model Number shapes Counters	Ten frames Bead strings Number lines
Multiply 2-digit by 1-digit numbers	3/4	Place value counters Base 10	Expanded written method Short written method
Multiply 3-digit by 1-digit numbers	4	Place value counters Base 10	Short written method
Multiply 4-digit by 1-digit numbers	5	Place value counters	Short written method

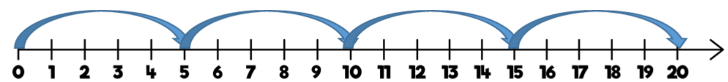
Skill	Year	Representations and models	
Multiply 2-digit by 2-digit numbers	5	Place value counters Base 10	Short written method Grid method
Multiply 2-digit by 3-digit numbers	5	Place value counters	Short written method Grid method
Multiply 2-digit by 4-digit numbers	5/6	Formal written method	

Skill: Solve 1-step problems using multiplication	Year: 1/2

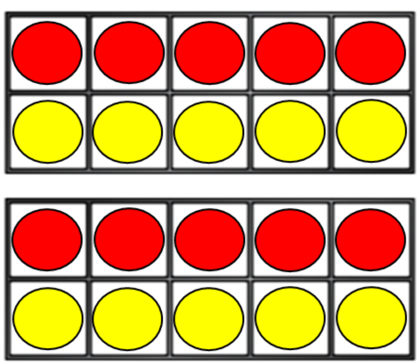


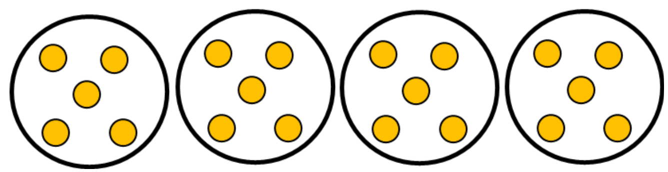


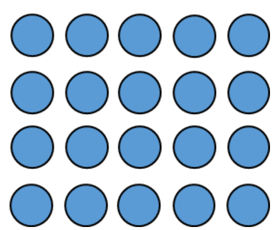




One bag holds 5 apples.
How many apples do 4 bags hold?







$5 + 5 + 5 + 5 = 20$

$4 \times 5 = 20$

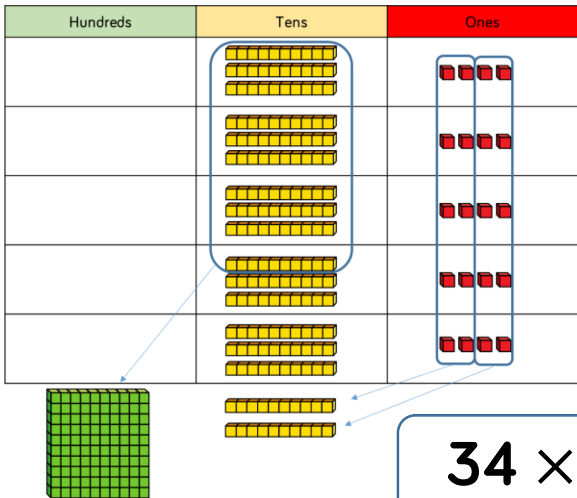
$5 \times 4 = 20$

Children represent multiplication as repeated addition in many different ways.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.

In Year 2, children are introduced to the multiplication symbol.

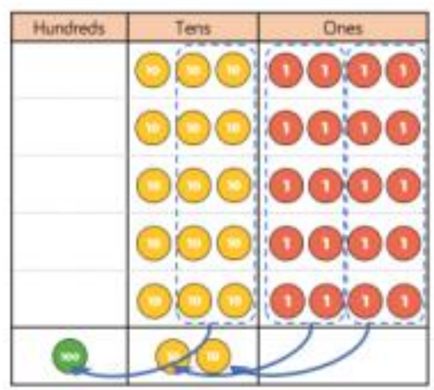
Skill: Multiply 2-digit numbers by 1-digit numbers	Year: 3/4



	H	T	O		
		3	4		
×			5		
		2	0	(5 × 4)	
+	1	5	0	(5 × 30)	
	1	7	0		

$$34 \times 5 = 170$$

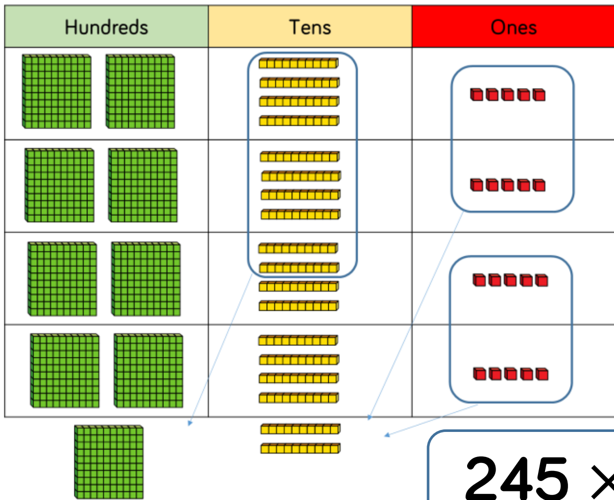
	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		



Informal methods and the expanded method are used in Year 3 before moving on to the short multiplication method in Year 4.

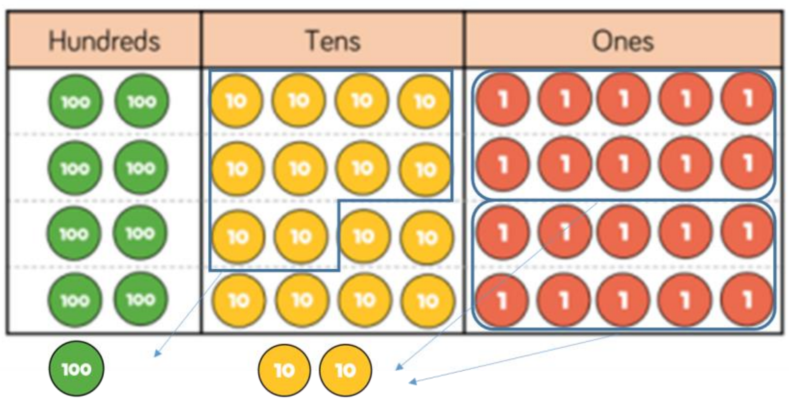
Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

Skill: Multiply 3-digit numbers by 1-digit numbers	Year: 4



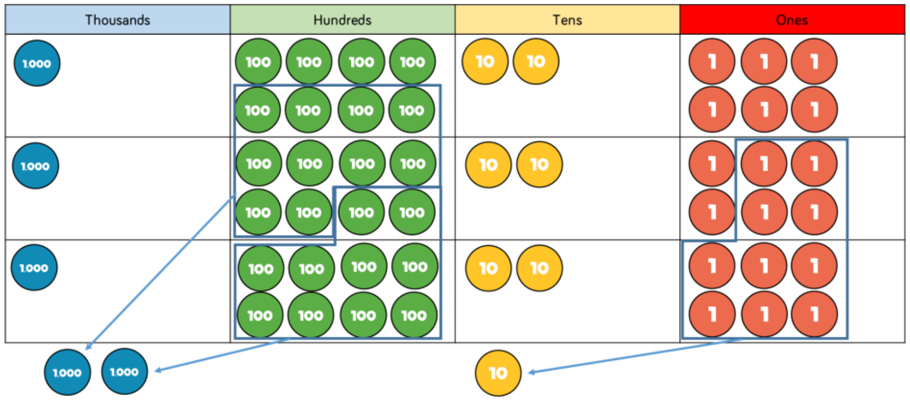
	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

$$245 \times 4 = 980$$



When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method.
Base 10 and place value counters continue to support the understanding of the written method.
Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

Skill: Multiply 4-digit numbers by 1-digit numbers	Year: 5

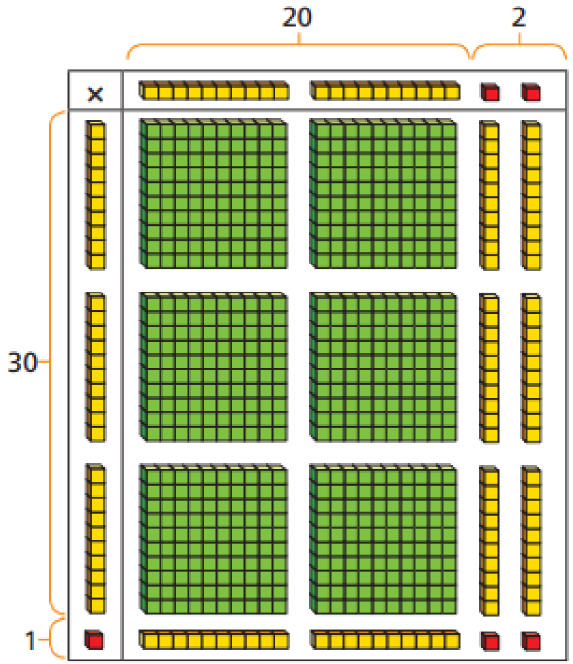


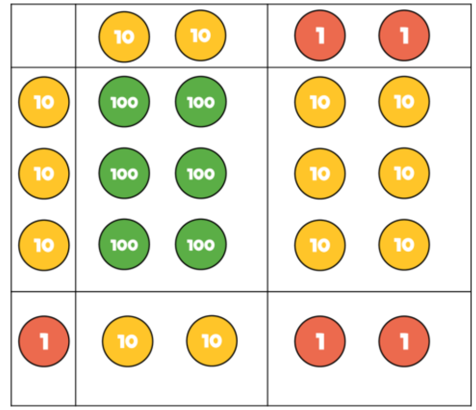
$$1,826 \times 3 = 5,478$$

	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method.
If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

Skill: Multiply 2-digit numbers by 2-digit numbers	Year: 5





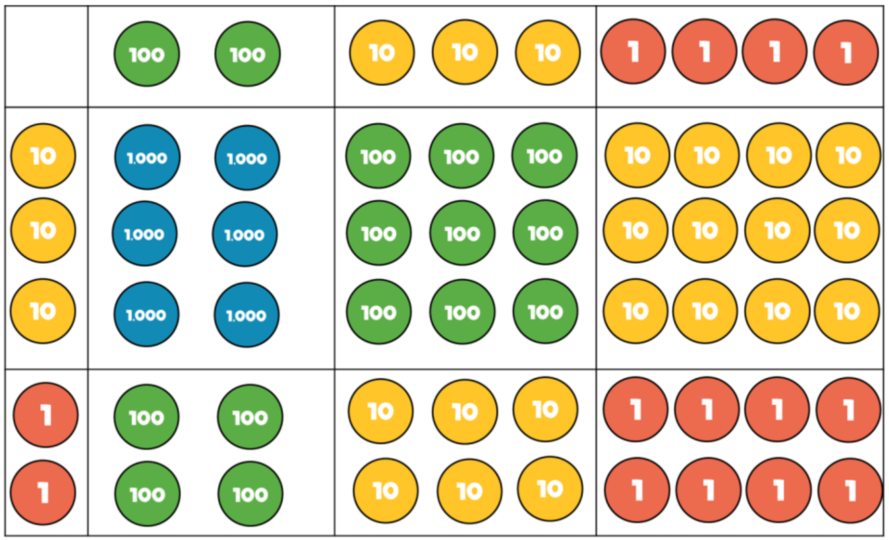
×	20	2
30	600	60
1	20	2

	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

$22 \times 31 = 682$

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

Skill: Multiply 3-digit numbers by 2-digit numbers	Year: 5



Th	H	T	O
	2	3	4
×		3	2
	4	6	8
1 7	1 0	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

234 × 32 = 7,488

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Children should now move towards the formal written method, seeing the links with the grid method.

Skill: Multiply 4-digit numbers by 2-digit numbers	Year: 5/6

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 2	1 5	9 3	1 7	2
5 1	4	7 1	8	0
7	6	6	9	2

1

$2,739 \times 28 = 76,692$

When multiplying 4-digits by 2-digits, children should be confident in using the formal written method.

If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

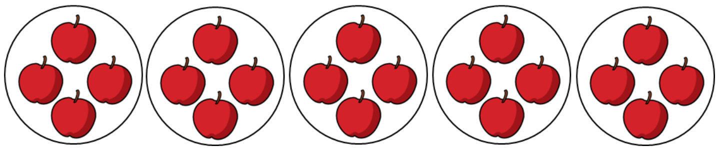
Division

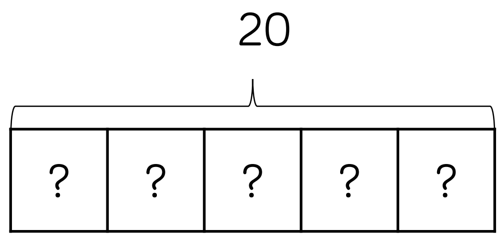
Skill	Year	Representations and models	
Solve one-step problems with division (sharing)	1/2	Bar model Real life objects	Arrays Counters
Solve one-step problems with division (grouping)	1/2	Real life objects Number shapes Bead strings Ten frames	Number lines Arrays Counters
Divide 2-digits by 1-digit (no exchange sharing)	3	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with exchange)	3	Straws Base 10 Bar model	Place value counters Part-whole model

Skill	Year	Representations and models	
Divide 2-digits by 1-digit (sharing with remainders)	3/4	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1-digit (sharing with exchange)	4	Base 10 Bar model	Place value counters Part-whole model
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division

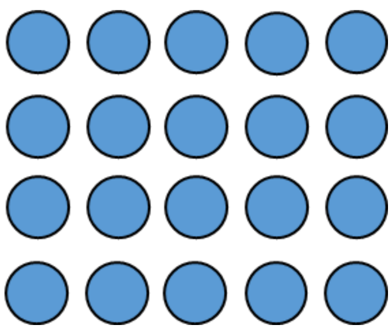
Skill	Year	Representations and models	
Divide 4-digits by 1-digit (grouping)	5	Place value counters Counters	Place value grid Written short division
Divide multi-digits by 2-digits (short division)	6	Written short division	List of multiples
Divide multi-digits by 2-digits (long division)	6	Written long division	List of multiples

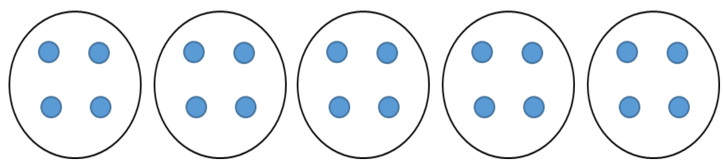
Skill: Solve 1-step problems using multiplication (sharing)	Year: 1/2





There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?





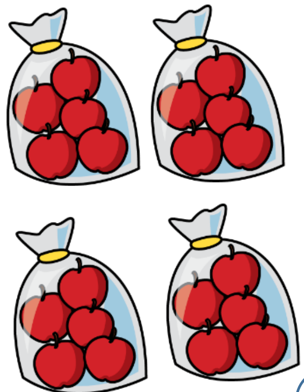
$20 \div 5 = 4$

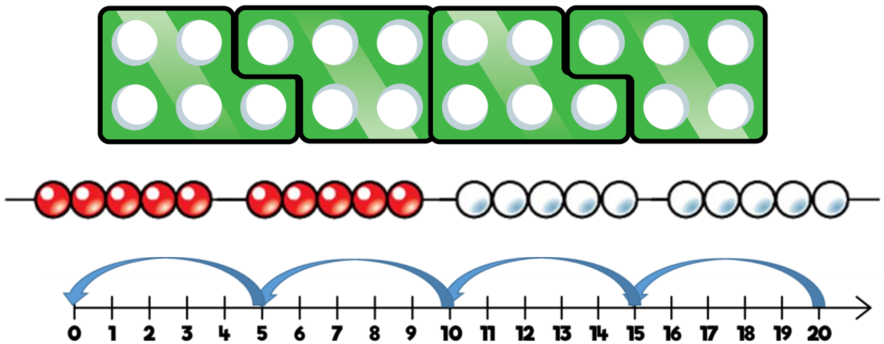
Children solve problems by sharing amounts into equal groups.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.

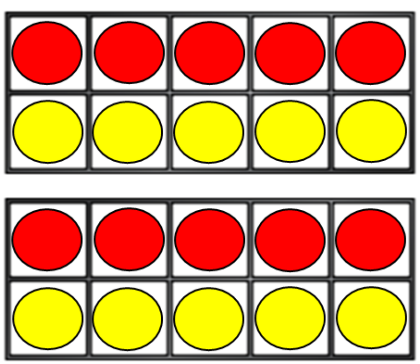
In Year 2, children are introduced to the division symbol.

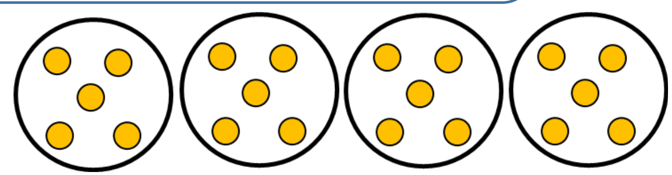
Skill: Solve 1-step problems using division (grouping)	Year: 1/2

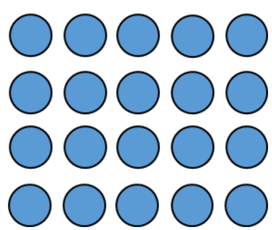




There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?



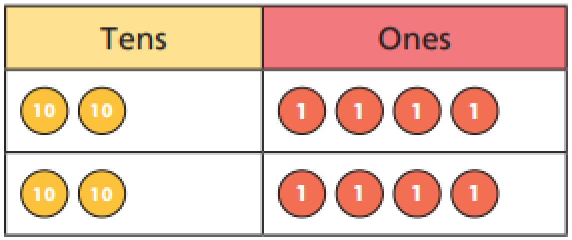


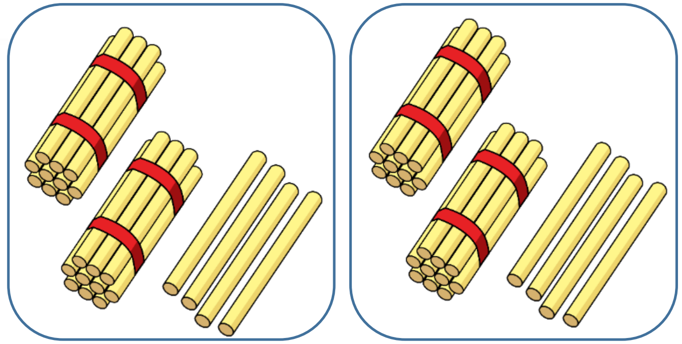


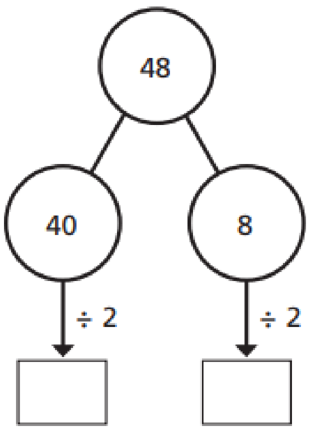
$20 \div 5 = 4$

Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

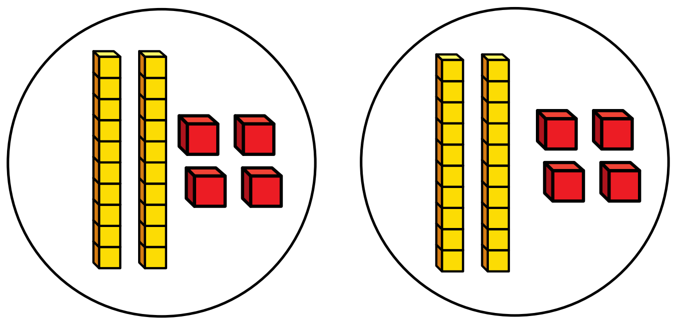
Skill: Divide 2-digits by 1-digit (sharing with no exchange)	Year: 3







$$48 \div 2 = 24$$

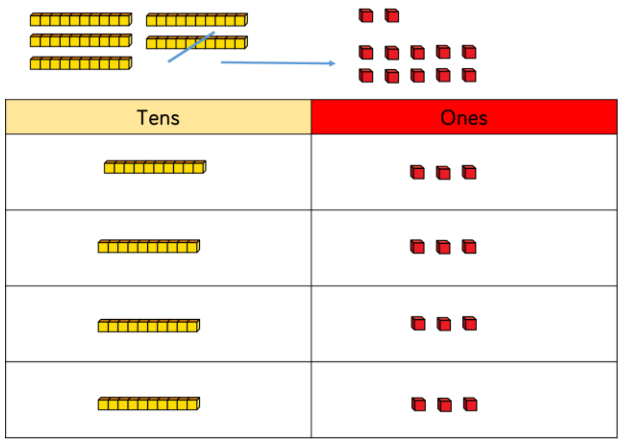


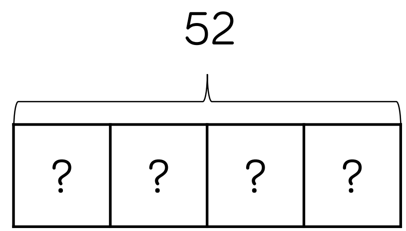
When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

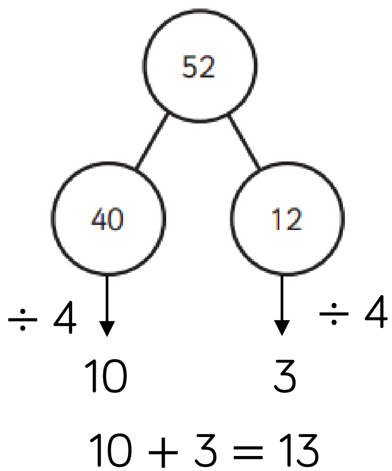
Part-whole models can provide children with a clear written method that matches the concrete representation.

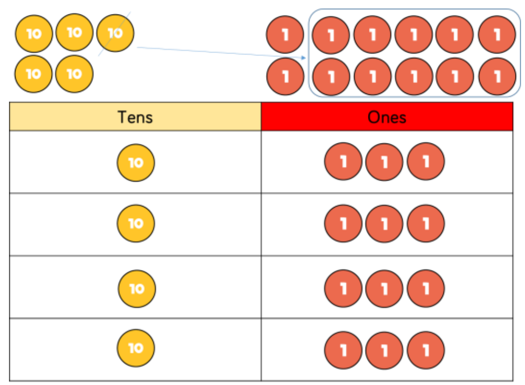
Skill: Divide 2-digits by 1-digit (sharing with exchange)	Year: 3/4





$$52 \div 4 = 13$$

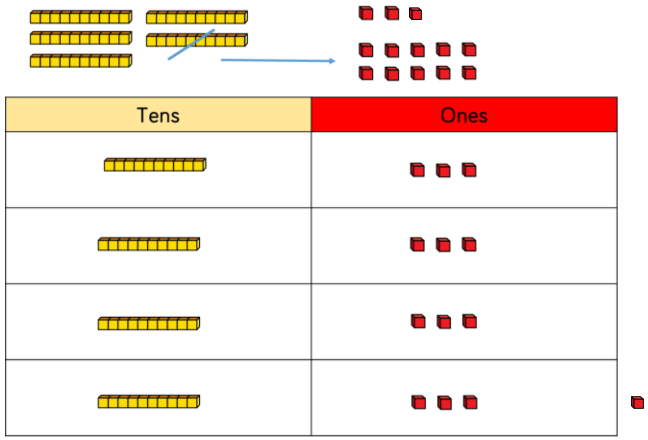


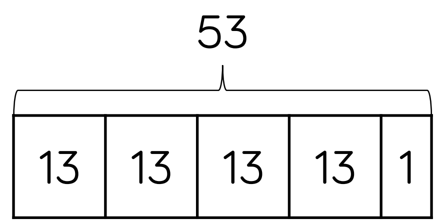


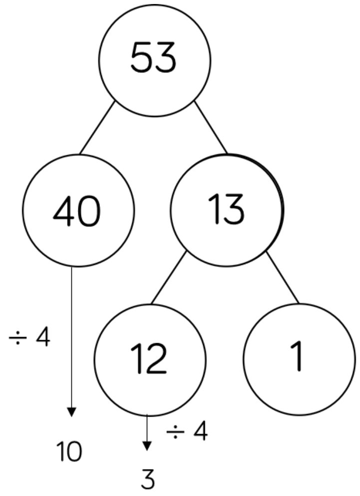
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model supports this method.

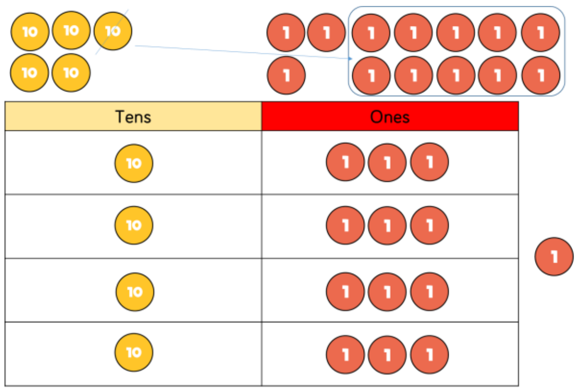
Skill: Divide 2-digits by 1-digit (sharing with remainders)	Year: 3/4







$$53 \div 4 = 13 \text{ r1}$$

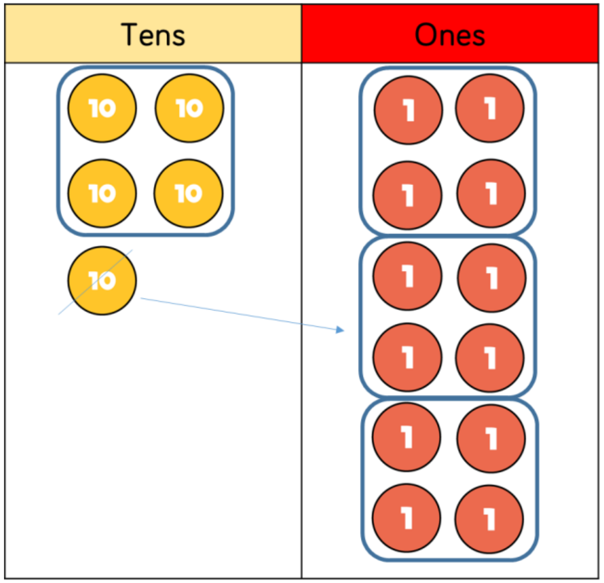


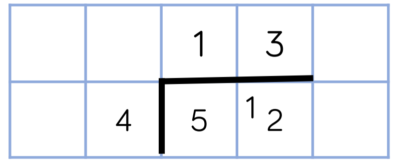
When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones.

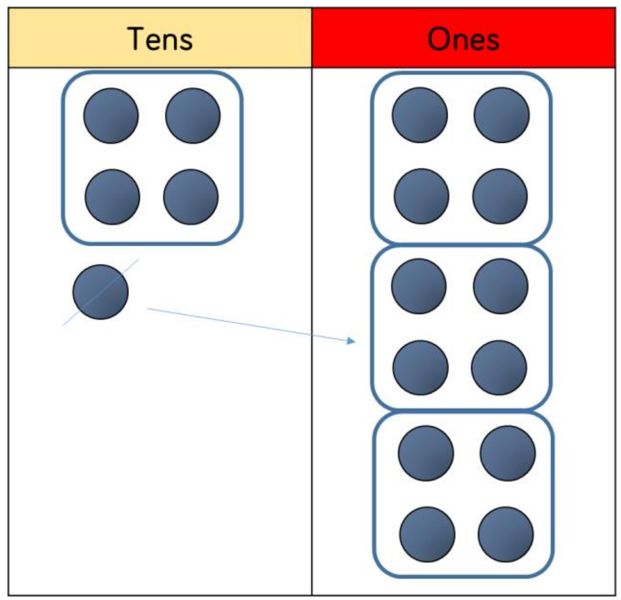
Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made.

Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (grouping) | Year: 5







$$52 \div 4 = 13$$

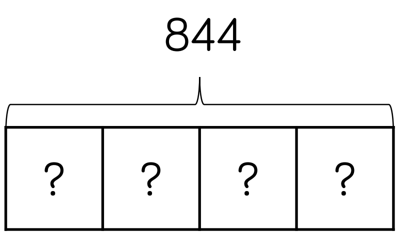
When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

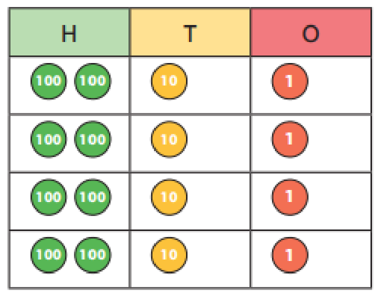
Language is important here. Children should consider ‘How many groups of 4 tens can we make?’ and ‘How many groups of 4 ones can we make?’

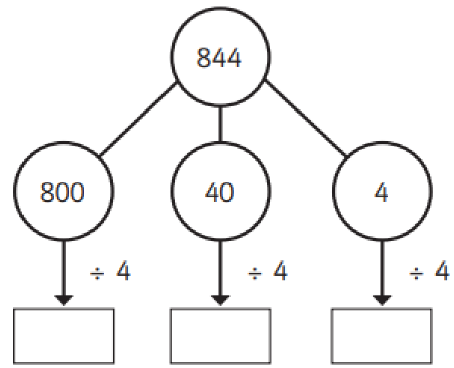
Remainders can also be seen as they are left ungrouped.

Skill: Divide 3-digits by 1-digit (sharing)	Year: 4

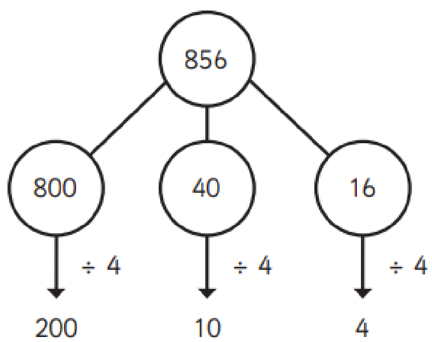
$844 \div 4 = 211$

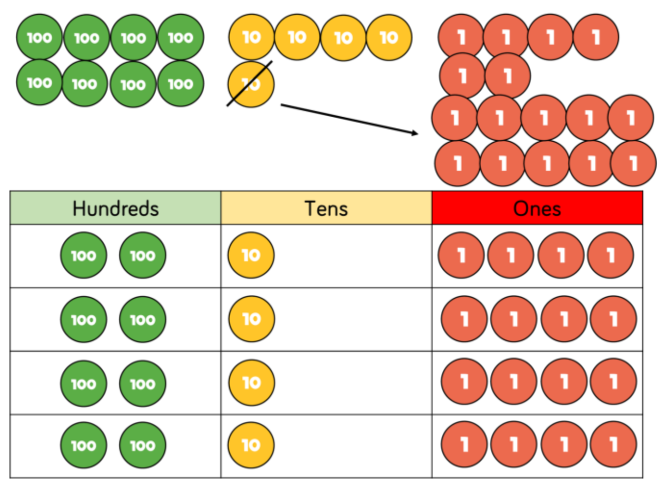






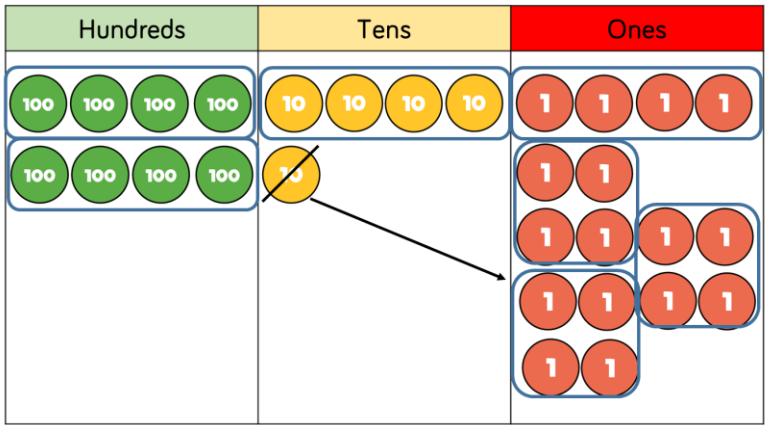
$856 \div 4 = 214$



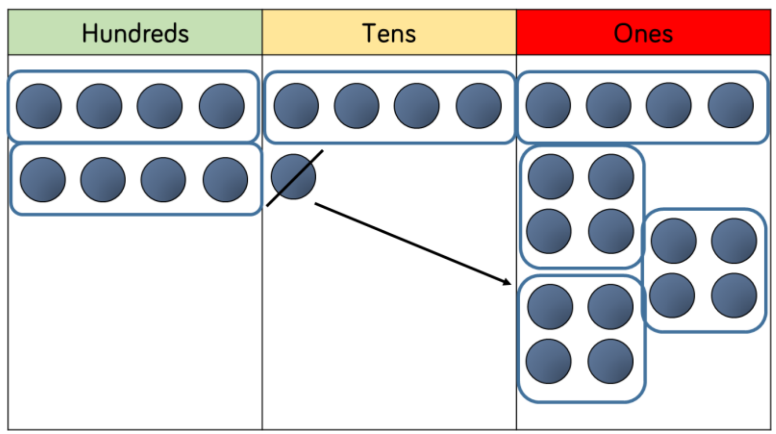


Children can continue to use place value counters to share 3-digit numbers into equal groups.
Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows.
This method can also help to highlight remainders.
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 3-digits by 1-digit (grouping)	Year: 5



		2	1	4
	4	8	5	$^{1}6$

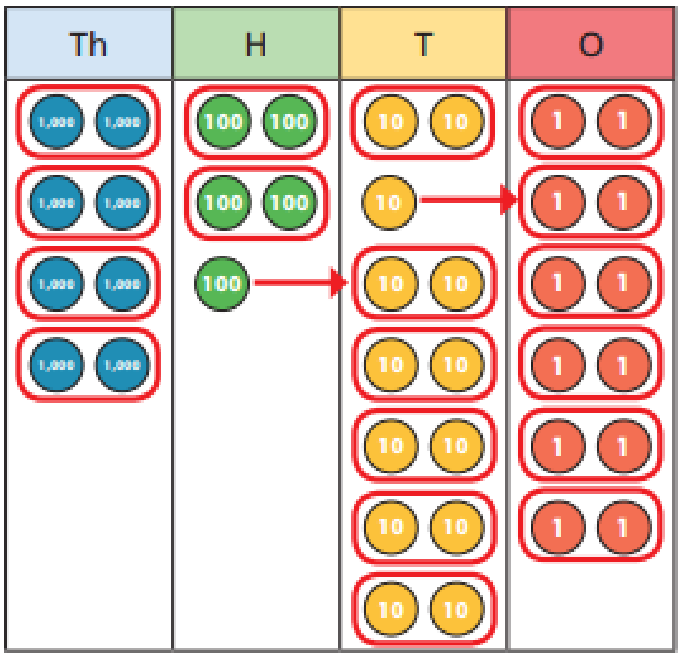


$856 \div 4 = 214$

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.

Skill: Divide 4-digits by 1-digit (grouping)	Year: 5



	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

$$8{,}532 \div 2 = 4{,}266$$

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method.

Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

Skill: Divide multi digits by 2-digits (short division)	Year: 6

		0	3	6
	12	4	$^{4}3$	$^{7}2$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi-digits by 2-digits (long division)	Year: 6
	Children can also divide by 2-digit numbers using long division. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

		0	3	6	
1	2	4	3	2	
	−	3	6	0	(×30)
			7	2	
	−		7	2	(×6)
				0	

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9	
15	7	3	3	5	
−	6	0	0	0	(×400
	1	3	3	5	
−	1	2	0	0	(×80)
		1	3	5	
−		1	3	5	(×9)
				0	

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Skill: Divide multi digits by 2-digits (long division)	Year: 6
$372 \div 15 = 24\ r12$	When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.
$372 \div 15 = 24\frac{4}{5}$	Children can also answer questions where the quotient needs to be rounded according to the context.

			2	4	r	1	2
1	5	3	7	2			
	−	3	0	0			
			7	2			
	−		6	0			
			1	2			

$1 \times 15 = 15$

$2 \times 15 = 30$

$3 \times 15 = 45$

$4 \times 15 = 60$

$5 \times 15 = 75$

$10 \times 15 = 150$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	−	3	0	0	
			7	2	
	−		6	0	
			1	2	

Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor

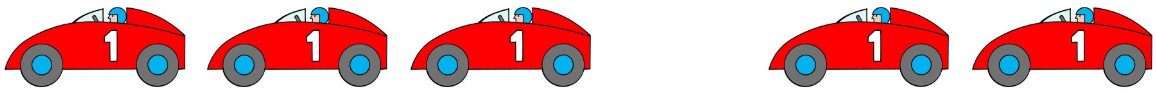
SUPPLEMENTARY GUIDANCE

UNDERSTANDING IN MATHS

"The success of an activity is based on whether it represents the maths in a way the child understands." **J.Bruner**

1) **ENACTIVE (concrete)** —internalised *action* with objects eg. moving 3 cars and 2 cars beside each other.

e.g.



2) **ICONIC (pictorial)** —sensory imagery or pictures.

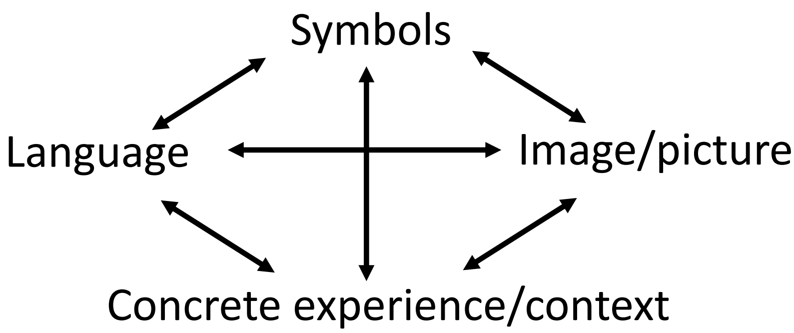
e.g.

All learning in mathematics should transition through these three stages to secure understanding.

3) **SYMBOLIC (abstract)** —arbitrary symbols, talk.

e.g. 'three plus two' or '3+2'

"The teacher's role in developing understanding is… to help the child build up connections between new experiences and previous learning." **Haylock & Cockburn**



"...Pupils should make connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems..." **National Curriculum**

"Schools should develop the expertise of staff: In choosing teaching approaches and activities that foster pupils' deeper understanding, I checking and probing pupils' understanding during the lesson, in understanding the progression in strands of maths over time, so that they know the key knowledge and skills that underpin each stage of learning..." **OFSTED**

PROMPTING CHILDREN'S THINKING THROUGH QUESTIONING

The following key questions, ideas and strategies should be used in maths lessons to deepen understanding and prompt thinking/reasoning skills:

What do you notice? What's the same? What's different?

- Spot the mistake/Which is correct?
- True or false?
- What comes next?
- Do, then explain
- Make up an example/Write more statements
- Possible answers/Other possibilities
- Continue the pattern
- Missing numbers/symbols/information
- Working backwards/Use the inverse/ Undoing
- Hard and easy questions
- What else do you know?/Use a fact
- Fact families
- Convince me/Prove it/Generalising
- Make an estimate/Size of an answer
- Always, sometimes, never
- Making links/Application
- Can you find?
- Odd one out
- Complete the pattern/Continue the pattern
- Another and another
- Ordering
- Testing conditions
- The answer is…
- Visualising

NCETM 2015 Reasoning Strategies/ Pedagogies

It is important to ensure that there are lots of opportunities for maths talk to happen throughout the lesson.

As well as the above questions try to include the following key ideas:

- Enabling learning through:
  - > **Drawing attention to...**
  - **> Developing reasoning and making connections.**
- Providing opportunities for children to:
  - **> Manipulate, experience, see**
  - **> Engage in talk (listen, analyse and discuss)**
- Developing children's thinking through:
  - **> Investigation**
  - **> Scaffolding**

Opportunities in maths learning should include:

- **MATHEMATICAL THINKING—**reasoning to apply understanding and skills to solve problems.
- **PROPORTIONALITY–** understanding size and relationships of one thing to another.
- **PATTERN–** the structure and relationships in mathematical concepts.
- **GENERALITY–** connecting structures and relationships to apply rules.
- **REPRESENTATION–** representing questions in a variety of ways (enactive/ iconic/symbolic)

PLACE VALUE

The Curriculum has a strong emphasis on place value understanding and mastery.

- All numbers can be represented using the ten digits 0,1,2,3,4,5,6,7,8,9
- The value that each digit represents in determined by its place
- The principle of exchange—that when you have accumulated then in one place, it can be exchanged for one in the next place to the left.
- Connect how we say words and how we write them. (Nb. 'teen numbers)
- Understand how 0 is used as a place holder.
- Connect the symbols for numbers with the numberline.

EXCHANGE

'ten' 10 is the most important in our naming system because when we are counting collections, as soon as we have a group of ten, we call them something else.

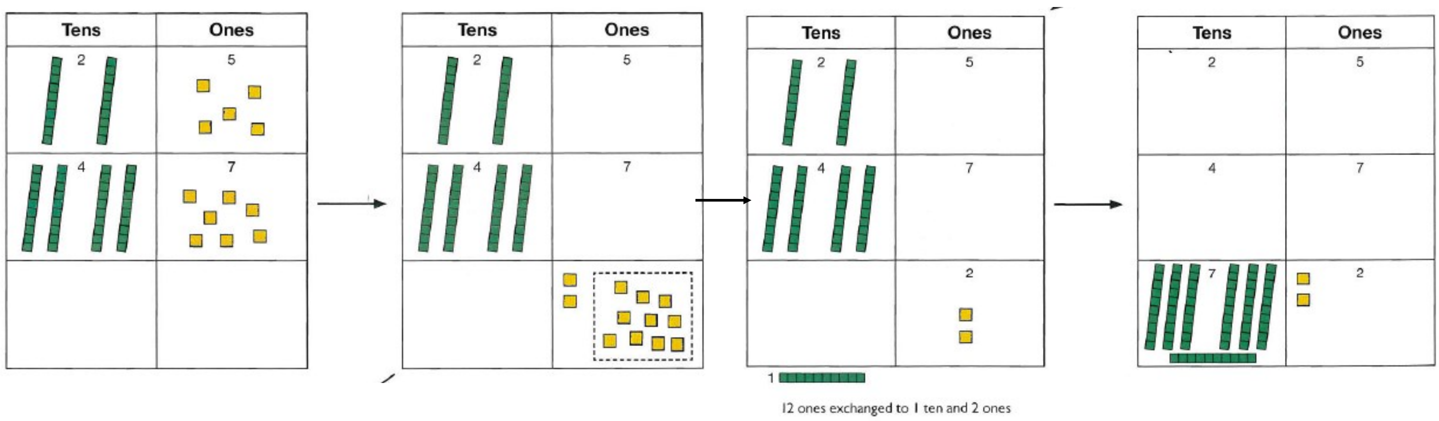
So:

ten *ones* are called one *ten* 1 1 1 1 1 1 1 1 1 1 = 10

ten *tens* are called one *hundred* 10 10 10 10 10 10 10 10 10 10 = 100

ten *hundreds* are called one *thousand* 100 100 100 100 100 100 100 100 100 100 = 1000

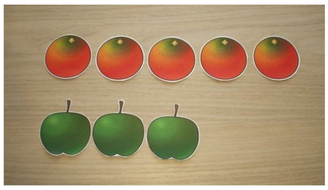
25 + 47

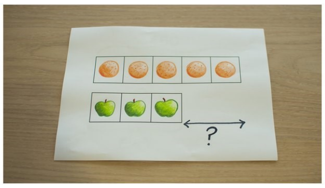


PROGRESSION OF BAR MODELLING

1) Imagine you have five oranges and three apples, how many more oranges than apples? (Concrete with actual objects)

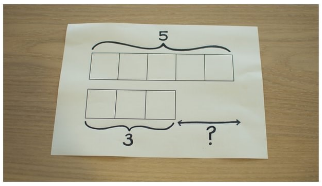
2) At first children model the problem with physical objects they can move around: like these cut-out pictures.

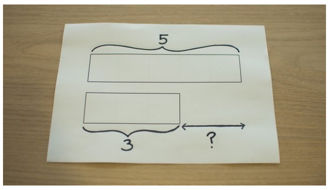




3) After a few months they start to draw pictures of the problem to help them think about it.

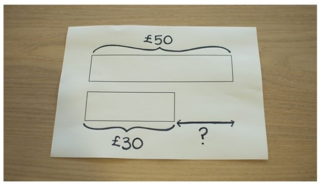
4) Over time children drop the pictures and just draw boxes. Then they start adding numbers as labels.

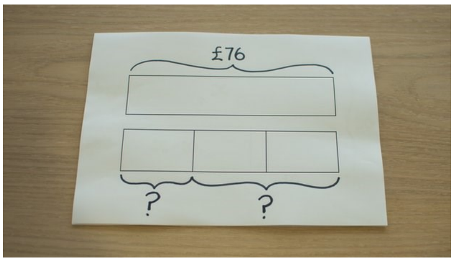




5) Once children are confident with the meaning of the number symbol they no longer need to draw all the boxes. However they know they can always draw the boxes in again if they need to convince themselves.

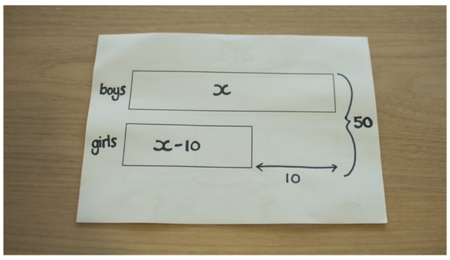
6) How much change if you pay for a £30 shirt with a £50 note? The model can be used to help visualise almost any maths problem.





7) Three people want to split a restaurant bill of £76. How much for a couple who want to pay together? The model helps break the problem down. First divide £76 by 3. Then times the answer by 2.

8) In a year group there are 50 children. There are 10 fewer girls than boys. How many boys? The model can help visualise the unknown quantity. You can see that x + x - 10 = 50. If you add the 10 you get x + x = 60. So x = 30



TIMES TABLES

https://www.youtube.com/watch?v=yXdHGBfoqfw

There is an increased emphasis on knowledge and recall of times-tables, with all children required to know up to 12x12 by the end of year 4.
Thinking back to the three stages of learning in maths—enactive, iconic and symbolic; are we presenting times-tables in these different ways to support learning/understanding rather than just recall?

The numberline technique used in the video above is working with the symbolic numbers but also supports this with the iconic representation of the number line via the counting stick.

Times tables practice should ensure that children have moved through these three stages of learning, rather than just chanting alone. When vocalising times tables, children should be able to say them in different ways eg. Listing '3,6,9,12' as well as full '1 times 3 is three, 2 times three is 6'. Also encourage different language use eg. '1 multiplied by 3' or 'the product of 1 and 3'.

<u>REMEMBER TO USE ARRAYS</u>

Arrays are one of the crucial steps from enactive to iconic in times tables—they allow the children to make important links in maths, for example seeing multiplications as repeated addition and division as a direct inverse of multiplication (eg. Sharing out the chocolates from the box!)